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A Proof of Thurston's Uniformization Theorem of Geometric Orbifolds

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§1. The uniformization theorem.

A smooth m-dimensional orbifold (briefly, an m-orbifold) is a σ-compact Hausdorff space M which is locally modelled on a quotient space of a finite group action on a smooth m-dimensional manifold ([Sa], [Th]). More precisely, an *m-orbifold M* is covered by an *atlas* of *folding charts* $\{(\tilde{U}_i, G_i, f_i, U_i)\}_{i\in I}$, each chart consisting of a smooth connected m-manifold $\tilde{U}_i$, a finite group G_i acting on $\tilde{U}_i$ smoothly and effectively, an open set U_i of M and a *folding map* $f_i : \tilde{U}_i \to U_i$ which induces a natural homeomorphism $G_i\backslash\tilde{U}_i \to U_i$. These charts must satisfy a certain compatibility condition. In a simplified version due to Bonahon and Siebenmann [BS], the condition states the following: for every $x\in\tilde{U}_i$ and $y\in\tilde{U}_j$ such that $f_i(x)=f_j(y)\in U_i\cap U_j$, there exists a diffeomorphism $\psi : \tilde{V}_x \to \tilde{V}_y$ from an open neighborhood of x in $\tilde{U}_i$ to an open neighborhood of y in $\tilde{U}_j$ such that $\psi(x)=y$ and $f_j\psi=f_i$. (For an explanation of this compatibility condition, see Appendix A.)

Two atlases on an m-orbifold give the same orbifold structure iff their union is again a compatible atlas.

For instance, let Γ be a group acting on a manifold $\tilde{U}$ smoothly, effectively and properly discontinuously. Then the quotient space $\Gamma\backslash\tilde{U}$ has the structure of a smooth orbifold. This type of an orbifold is said to be *good*.

The notion of being good is defined more formally in terms of orbifold coverings as follows. Let $h : N \to M$ be a continuous map of a connected orbifold N onto another orbifold M. h is called an *orbifold covering* of M if M admits an atlas of folding charts $\{(\tilde{U}_i, G_i, f_i, U_i)\}_{i\in I}$ such that, for each component V of $h^{-1}(U_i)$, there exists a folding chart $k : \tilde{U}_i \to V$ in the maximal atlas of N so that $f_i=hk$.

DEFINITION ([Th]). A connected orbifold M is *good* if there exists an orbifold

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