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Generalized Spencer Cohomology Groups and Quasi-Regular Bases

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Dedicated to Professor Tadashi Nagano on his 60th birthday

Introduction. Let $\mathfrak{g}=\bigoplus_{p\geq-\mu}\mathfrak{g}_p$ be a transitive graded Lie algebra of depth $\mu>0$, and let $\mathfrak{g}_-=\bigoplus_{p<0}\mathfrak{g}_p$ be its negative part. Associated with the adjoint representation of $\mathfrak{g}_-$ on $\mathfrak{g}$, there is defined a cohomology group $H(\mathfrak{g}_-,\mathfrak{g})=\bigoplus_{p,r}H^p(\mathfrak{g}_-,\mathfrak{g})_r$ which we call the generalized Spencer cohomology group (see §6).

If $\mu=1$, $\mathfrak{g}_-$ is abelian and the cohomology group is well known as the Spencer cohomology groups (see e.g., [2], [6]). The generalized Spencer cohomology group was introduced by Tanaka [7] and has been used extensively in our studies of filtered Lie algebras [3], geometric structures [4] and differential equations [5], based on filtered manifolds, where it is this cohomology group that takes the rôle of the Spencer cohomology group.

We know that $H^p(\mathfrak{g}_-,\mathfrak{g})_r$ vanishes for large r by Noetherian property (see [3]), however in various concrete problems we need further to compute this cohomology group explicitly or to determine the range of (p, r) in which $H^p(\mathfrak{g}_-,\mathfrak{g})_r$ vanishes. In the case $\mu=1$, as is well known, there is a fundamental theorem conjectured by Guillemin and Sternberg and proved by Serre (see Appendix of [2]), which relates the vanishing of the cohomology group with the existence of a quasi-regular basis.

The main purpose of this paper is to extend the theorem of Serre to the generalized Spencer cohomology group. We shall give a criterion (explicit and in some extent calculable) in terms of quasi-regular bases for the vanishing of the generalized Spencer cohomology group, and also make clear the difference which lies between the special case $\mu=1$ and the general case $\mu\geq1$.

The nature of our problem being better adapted to its dualized form, we shall mainly discuss homology groups of graded modules, and in the last section we translate the main results to the cohomology groups of graded Lie algebras.

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