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Curvature Functions for the Sphere in Pseudohermitian Geometry

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Dedicated to Professor Tadashi Nagano on his sixtieth birthday

1. Introduction.

In Jerison and Lee's work on the CR Yamabe problem [JL], they consider the following equation of prescribing pseudohermitian scalar curvature $(2(n+1)/n)R$ under the choice of contact forms in a fixed CR structure:

$$\Delta_b u+\frac{n}{2(n+1)}R_0 u-Ru^{(n+2)/n}=0\,,\qquad u>0 \tag{1.1}$$

with $R\equiv$constant, R_0 is a given pseudohermitian scalar curvature, where the sub-laplacian operator Δ_b is the real part of Kohn's $\Box_b$ acting on functions. (See §2 for the definition.)

Let S^{2n+1} be the unit sphere in C^{n+1} equipped with the canonical pseudohermitian structure having pseudohermitian scalar curvature $n(n+1)/2$ (see §2). In this paper, we study the problem of prescribing arbitrary R on S^{2n+1} with $R_0=n(n+1)/2$ in (1.1). In fact, the equation we consider reads

$$\Delta_b u+\frac{n^2}{4}u-Ru^a=0\,,\qquad u>0 \tag{1.2}$$

on S^{2n+1}, where $a>1$ is a constant. Our canonical pseudohermitian structure is determined by a certain contact form θ. Let L_θ denote the associated Levi form. The volume form $\theta\wedge(d\theta)^n$ is denoted by dv_θ. The gradient operator relative to the metric $\langle\ ,\ \rangle=(1/4)\theta^2+L_\theta$ is denoted by ∇. In §3, we obtain an integrability condition as follows.

THEOREM A. *If u is a positive solution of* (1.2), *then*

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