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Interpolation between Some Banach Spaces in Generalized Harmonic Analysis: The Real Method

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Introduction.

In [3], A. Beurling introduced the space $A^p(\boldsymbol{R}^1)$, $1<p<\infty$, as

$$A^p(\boldsymbol{R}^1)=\left\{f : \|f\|_{A^p(\boldsymbol{R}^1)}=\inf_{\omega\in\Omega}\left(\int_{-\infty}^{\infty}|f(x)|^p\omega(x)^{-(p-1)}dx\right)^{1/p}<\infty\right\},$$

where Ω is the class of functions ω on $\boldsymbol{R}^1$ such that ω is positive, even, nonincreasing with respect to $|x|$, and

$$\omega(0)+\int_{-\infty}^{\infty}\omega(x)dx=1 .$$

By regarding $A^p(\boldsymbol{R}^1)$ as an $L^1(\boldsymbol{R}^1)$ analog, Y. Chen and K. Lau [5] developed the H^1-theory analog. In particular, the maximal function characterization, the atomic decomposition, and the duality corresponding to Fefferman-Stein's H^1-BMO duality were shown. The $\boldsymbol{R}^n$ case was investigated by J. Garcia-Cuerva [6].

Recently, by regarding $A^p(\boldsymbol{R}^n)$ as an $L^p(\boldsymbol{R}^n)$ analog, K. Matsuoka [7] characterized the complex interpolation space $(A^{p_0}(\boldsymbol{R}^n), A^{p_1}(\boldsymbol{R}^n))_{[\theta]}$. His result is

$$(A^{p_0}(\boldsymbol{R}^n), A^{p_1}(\boldsymbol{R}^n))_{[\theta]}=(A^{p_0}(\boldsymbol{R}^n), A^{p_1}(\boldsymbol{R}^n))^{[\theta]}=A^p(\boldsymbol{R}^n) \qquad \text{(equal norms)},$$

where $1<p_0, p_1<\infty, 0<\theta<1, 1/p=(1-\theta)/p_0+\theta/p_1$. On the other hand, in the harmonic analysis, many real interpolation spaces have been studied by various authors: e.g.,

$$(L^{p_0}(\boldsymbol{R}^n), L^{p_1}(\boldsymbol{R}^n))_{\theta,p}=L^p(\boldsymbol{R}^n) \qquad \text{(equivalent quasi-norms)},$$

where $0<p_0, p_1<\infty$, $0<\theta<1$, $1/p=(1-\theta)/p_0+\theta/p_1$ (cf. J. Bergh and J. Löfström [2]).

In this paper, we will calculate the real interpolation space $(A^{p_0}(\boldsymbol{R}^n), A^{p_1}(\boldsymbol{R}^n))_{\theta,p}$, where $1<p_0, p_1<\infty$, $0<\theta<1$, $1/p=(1-\theta)/p_0+\theta/p_1$, and also show the related interpolation results.

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