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Compact Weighted Composition Operators on Certain Subspaces of $C(X, E)$

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§1. Introduction and results.

Let X be a compact Hausdorff space and E a complex Banach space with the norm $\|\cdot\|_E$. By $C(X, E)$ we denote the Banach space of all continuous E-valued functions on X with the usual norm; $\|f\|=\sup\{\|f(x)\|_E : x\in X\}$. When E is the complex field $\boldsymbol{C}$, we use $C(X)$ in place of $C(X, \boldsymbol{C})$. Let A be a function algebra on X, that is, a closed subalgebra of $C(X)$ which contains the constants and separates points of X. We define the space $A(X, E)$ by

$$A(X, E)=\{f\in C(X, E) : e^*\circ f\in A \text{ for all } e^*\in E^*\},$$

where E^* is the dual space of E. Clearly $A(X, E)$ is a Banach space relative to the same norm. For example, as a generalization of the disc algebra $A(\bar{D})$ on the closed unit disc $\bar{D}$, we may consider the space $\{f\in C(\bar{D}, E) : f$ is an analytic E-valued function on the open unit disc $D\}$. Here f is said to be analytic on D when it is differentiable at each point of D, in the sense that the limit of the usual difference quotient exists in the norm topology. It is known that this space coincides the following space;

$$\{f\in C(\bar{D}, E) : e^*\circ f\in A(\bar{D}) \text{ for all } e^*\in E^*\}$$

(see [2, p. 126]). The above definition of $A(X, E)$ is abstracted from this property.

We investigate weighted composition operators on $A(X, E)$. A weighted composition operator on $A(X, E)$ is a bounded linear operator T from $A(X, E)$ into itself, which has the form;

$$Tf(x)=w(x)f(\varphi(x)), \qquad x\in X,\ f\in A(X, E),$$

for some selfmap φ of X and some map w from X into $B(E)$, the space of bounded linear operators on E. We write wC_φ in place of T.

Weighted composition operators or composition operators on $C(X, E)$ were studied in [3] and [6], and the case of $E=\boldsymbol{C}$ was considered by Kamowitz [4], Uhlig [8],

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