

Hölder Continuity of Sample Paths of Some Self-Similar Stable Processes

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Dedicated to Professor Tatsuo Kawata on his eightieth birthday

1. Introduction and results.

A stochastic process $\{X(t)\}$ is said to be H -self-similar (H -ss) for $H > 0$ if for any $c > 0$, all finite-dimensional distributions of $\{X(ct)\}$ are the same as those of $\{c^H X(t)\}$, and to have stationary increments (si) if any finite-dimensional distribution of $\{X(t+b) - X(t)\}$ does not depend on b . It is also said to be α -stable if any finite-dimensional distribution of $\{X(t)\}$ is α -stable.

In this paper, we examine the Hölder continuity of H -ss si α -stable processes.

There are two main classes of H -self-similar α -stable processes with stationary increments: the linear fractional stable processes and the harmonizable fractional stable processes. In [T], Takashima showed the Hölder continuity of the linear fractional stable processes when $1 < \alpha < 2$ and $1/\alpha < H < 1$, and also pointed out that the exponent in the Hölder continuity cannot be bigger than $H - 1/\alpha$. However, we can get a better Hölder continuity for the harmonizable fractional stable processes as follows. The harmonizable fractional stable process is a complex-valued process defined by

$$X(t) = \int_{-\infty}^{\infty} \frac{e^{it\lambda} - 1}{i\lambda} |\lambda|^{1-H-1/\alpha} d\tilde{M}_\alpha(\lambda),$$

where $0 < H < 1$ and \tilde{M}_α is a complex rotationally invariant α -stable motion, (see [CM]). This is an H -ss si rotationally invariant α -stable process.

THEOREM 1. *Let $0 < H < 1$ and $0 < \alpha < 2$. For the harmonizable fractional stable process, there exists a version X^* such that*

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