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Finite Type Hypersurfaces of a Sphere

Yasuyuki NAGATOMO

Tokyo Metropolitan University
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§1. Introduction.

Let M be an n-dimensional compact submanifold of an m-dimensional Euclidean space $\boldsymbol{R}^m$ and Δ the Laplacian of M (with respect to the induced metric) acting on smooth functions on M. We denote by x the position vector of M in $\boldsymbol{R}^m$. Then we have the following spectral decomposition of x;

$$x=x_0+\sum_{t\geq 1} x_t \qquad \Delta x_t=\lambda_t x_t \qquad (\text{in } L^2\text{-sense})\,. \tag{1.1}$$

If there are exactly k nonzero x_t's ($t\geq 1$) in the decomposition (1.1), then the submanifold M is said to be of *k-type*. Here x_0 in (1.1) is exactly the center of mass in $\boldsymbol{R}^m$. A submanifold M of a hypersphere S^{m-1} of $\boldsymbol{R}^m$ is said to be *mass-symmetric* in S^{m-1} if the center of mass of M in $\boldsymbol{R}^m$ is the center of the hypersphere S^{m-1} in $\boldsymbol{R}^m$.

In terms of these notions, a well-known result of Takahashi (cf. [6]) says that a submanifold M in $\boldsymbol{R}^m$ is of 1-type if and only if M is a minimal submanifold of a hypersphere S^{m-1} of $\boldsymbol{R}^m$. Furthermore, a minimal submanifold of a hypersphere S^{m-1} in $\boldsymbol{R}^m$ is mass-symmetric in S^{m-1}. On the other hand, in [3], mass-symmetric, 2-type hypersurfaces of S^{m-1} are characterized. In [1], it is proved that a compact 2-type surface in S^3 is mass-symmetric.

In this paper, we will show that many 2-type hypersurfaces of a hypersphere S^{n+1} are mass-symmetric and that mass-symmetric, 2-type hypersurfaces of S^{n+1} have no umbilic point. More precisely, we will prove the following.

THEOREM 1. *Let $x: M\to S^{n+1}$ be a compact hypersurface of a hypersphere S^{n+1} in $\boldsymbol{R}^{n+2}$. If M is of 2-type (i.e., $x=x_0+x_p+x_q$) and*

$$(\lambda_p+\lambda_q)-\frac{9n+16}{(3n+2)^2}\lambda_p\lambda_q\geq n\,,$$

then M is mass-symmetric (i.e., $x_0=0$).

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