TOKYO J. MATH.
VOL. 14, No. 1, 1991

Positive Projections on C^*-Algebras

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(Communicated by J. Tomiyama)

§1. Introduction.

In recent years there has been considerable progress in the study of certain linear maps of C^*-algebras which preserve the natural partial ordering. Suppose A is a unital C^*-algebra and P a unital positive projection of A into itself. It is known [6, 22] that $P(A)$ is a C^*-algebra under the product $a \circ b = P(ab)$ if P is completely positive, and P is automatically completely positive if $P(A)$ is a C^*-algebra. E. Størmer [18] linked the decomposability of P, which is weaker than complete positivity, to the theory of JC-algebras. In [15], A. G. Robertson has showed that the decomposability of P is equivalent to the existence of decomposition of P as a sum of a 2-positive map and a 2-copositive map under some condition. The global structure of positive linear maps is, however, very complicated, even in the finite dimensional case [3, 4, 9, 18, 21].

In this paper we shall investigate the difference between complete positivity and positivity of contractive projections on C^*-algebras, particularly in case of matrix algebras. As an application, we shall describe general C^*-algebras for which n-positivity coincides with $(n+1)$-positivity in the class of projections.

Let A and B be C^*-algebras. We do not assume units for C^*-algebras. The $n \times n$ matrix space over A, that is, $M_n(A)$ naturally inherits the corresponding order as a C^*-algebra. A C^*-algebra is said to be n-subhomogeneous if every irreducible representation of the algebra is finite dimensional with dimension not greater than n. Let ϕ be a positive linear map of A into B. Recall that ϕ is said to be n-positive (respectively, n-copositive) if the n-multiplicity map $\phi(n)$ (respectively, the n-comultiplicity map $\phi^c(n)$),

$$\phi(n) : [a_{i,j}] \in M_n(A) \longrightarrow [\phi(a_{i,j})] \in M_n(B)$$

$$(\text{respectively, } \phi^c(n) : [a_{i,j}] \in M_n(A) \longrightarrow [\phi(a_{j,i})] \in M_n(B))$$

is positive. The map ϕ is completely positive if it is n-positive for every positive integer n. It is, however, known that every n-positive map on an n-subhomogeneous C^*-

Received May 9, 1990