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Cyclotomic Function Fields with Divisor Class Number One

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Introduction.

Let k be a rational function field over the finite field $\boldsymbol{F}_q$ with q elements. In 1974 D. R. Hayes [Ha1] constructed the maximal abelian extension of k developing an idea of Carlitz. In his construction he uses "cyclotomic" function fields which are closely analogous to classical cyclotomic extensions of the rational number field.

In this paper we give genus formulae for the maximal "real" subfields of cyclotomic function fields, and apply the formulae to the determination of cyclotomic function fields and their maximal real subfields with divisor class number one respectively.

1. Preliminaries.

In this section we provide a quick review of Carlitz-Hayes' theory.

Let k be as in Introduction. We fix a generator T of k such that $k=\boldsymbol{F}_q(T)$ and put $R=\boldsymbol{F}_q[T]$. We denote by ∞ the prime divisor corresponding to the pole of T i.e. $\mathrm{ord}_\infty(f)=-\deg(f)$ for $f\in R$. We define an action of R to the additive group of $\bar{k}$ an algebraic closure of k as follows. For any $u\in\bar{k}$,

$$u^T=u^q+Tu\,,$$

$$u^\alpha=\alpha u\qquad(\alpha\in\boldsymbol{F}_q)\,.$$

It is easily checked that this action endows $\bar{k}$ with an R-module structure. For an $M\in R$ we put

$$\Lambda_M=\{\lambda\in\bar{k}\,;\,\lambda^M=0\}$$

and $K_M=k(\Lambda_M)$. In the following we assume $M\in R\setminus\boldsymbol{F}_q$. Then we can prove the following facts, which are quite analogous to the case of the classical cyclotomic theory.

1. As a polynomial in u over k, u^M is separable of degree q^d, where d is the degree

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