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Local Rings of Cohen-Macaulay *F*-Rational Rings Are *F*-Rational

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1. Introduction.

Let p be a prime number and let R be a commutative Noetherian ring of $\mathrm{ch}\, R=p$. We put $R^0=R\setminus\bigcup_{\mathfrak{p}\in \mathrm{Min}\, R}\mathfrak{p}$. Then for each ideal I of R the tight closure I^* of I is defined as follows:

$$I^*:=\{x\in R \mid {}^{\exists}c\in R^0 \text{ such that } c\cdot x^{p^e}\in I^{[p^e]} \text{ for all } e\gg 0\}\,,$$

where $I^{[p^e]}$ denotes the ideal of R generated by the elements i^{p^e} $(i\in I)$. Notice that I^* is an ideal of R and

$$I\subset I^*\subset \bar{I}\,,$$

where $\bar{I}$ denotes the integral closure of I.

The notion of tight closure was introduced by Hochster and Huneke [3] and they are now developing a marvellous theory on tight closures. For example using it they gave a beautiful new proof of the Briançon-Skoda theorem in characteristic p. See [4] for the detail.

The purpose of the present paper is to prove the following

THEOREM (1.1). *Let R be a Cohen-Macaulay local ring of* $\mathrm{ch}\, R=p$ *and suppose that $Q^*=Q$ for some parameter ideal Q of R. Then for any $\mathfrak{p}\in\mathrm{Spec}\, R$ and for any parameter ideal J of $R_{\mathfrak{p}}$ we have $J^*=J$ in $R_{\mathfrak{p}}$.*

We say that a Noetherian local ring R of $\mathrm{ch}\, R=p$ is *F-rational* if $Q^*=Q$ for any parameter ideal Q of R (cf. [1]). With this terminology our theorem (1.1) guarantees that every local ring of a Cohen-Macaulay F-rational local ring is again F-rational. The ring R is called *F-regular* if $I^*=I$ in $R_{\mathfrak{p}}$ for any $\mathfrak{p}\in\mathrm{Spec}\, R$ and for any ideal I of $R_{\mathfrak{p}}$. When R is a Gorenstein local ring, it is proved in [3, Proposition 5.1] that $I^*=I$ for any ideal I of R once $Q^*=Q$ for some parameter ideal Q of R. Therefore as an immediate consequence of Theorem (1.1) we get

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