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On the Generalized Hilbert Transforms in R^2 and the Generalized Harmonic Analysis*

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Dedicated to Professor Tatsuo Kawata on his eightieth birthday

Introduction.

In [17, 18], N. Wiener established the generalized harmonic analysis (GHA) in order to give an account of phenomena which cannot be described by the classical harmonic analysis. Especially, his GHA was motivated by even earlier investigations in the theory of Brownian motion, in order to study the functions with continuous spectra. And, he studied the uniformly almost periodic functions in the sense of H. Bohr from the point of view of the GHA (cf. S. Koizumi [7, 8] and P. Masani [9–11]).

S. Koizumi [4] introduced the generalized Hilbert transform (GHT) of $g\in L_c^2(\boldsymbol{R})$, i.e. g such that $g(x)/(x+i)\in L^2(\boldsymbol{R})$, by

$$\tilde{g}(x)=\lim_{\varepsilon\to 0}\frac{x+i}{\pi}\int_{0<\varepsilon\le|x-t|}\frac{g(t)}{t+i}\frac{dt}{x-t}.$$

And, in [5, 6], he constructed the theory of the spectral analysis of the GHT by using Wiener's generalized Fourier transform (GFT)

$$s(u;g)=\underset{A\to\infty}{\text{l.i.m.}}\frac{1}{\sqrt{2\pi}}\left[\int_1^A+\int_{-A}^{-1}\right]g(t)\frac{e^{-iut}}{-it}dt+\frac{1}{\sqrt{2\pi}}\int_{-1}^{1}g(t)\frac{e^{-iut}-1}{-it}dt\qquad(g\in L_c^2(\boldsymbol{R}))\,,$$

where "l.i.m." means the limit in $L^2(\boldsymbol{R})$. Then he proved the following fundamental Wiener's GFT relation between any $g\in L_c^2(\boldsymbol{R})$ and its GHT $\tilde{g}$:

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