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On Morimoto Algorithm in Diophantine Approximation

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Introduction.

Let us denote the continued fraction expansion of an irrational number α $(0<\alpha<1)$ by

$$\alpha=[0: e_1, e_2, \cdots],$$

and its n-th convergent by p_n/q_n. We call the sequence of partial quotients $\{e_i : i=1, 2, \cdots\}$ the name of α associated with the simple continued fraction algorithm. The following theorems are well known.

THEOREM A. (1) (Galois) *α is a reduced quadratic irrational, that is, a quadratic irrational whose algebraic conjugate $\bar{\alpha}$ satisfies $\bar{\alpha}<-1$, iff the name of α is purely periodic.*

(2) (Lagrange) *α is a quadratic irrational iff the name of α is eventually periodic.*

(3) (Klein) *Let $\Gamma_{(\pm)}$ be a polygon jointing the lattice points (q_{2n-1}, p_{2n-1}), $n=1, 2, \cdots$ $((q_{2n}, p_{2n}), n=0, 1, \cdots$ for $\Gamma_-)$ in this order, then the polygons are approximating polygons of the line $L: \alpha x-y=0$, that is, $\Gamma_{(\pm)}$ satisfies the following properties*:

(i) *$\Gamma_{(\pm)}$ is a convex (concave) polygon, and*

(ii) *The domain D enclosed by Γ_+ and Γ_- in the first quadrant includes the half line $\alpha x-y=0$, $x\geqq 0$, and the domain D does not contain any lattice point.*

(4) (Lévy) *For almost all α, we have*

1) $\displaystyle\lim_{n\to\infty}\frac{1}{n}\log q_n=\frac{\pi^2}{12\log 2}$ *and*

2) $\displaystyle\lim_{n\to\infty}\left(-\frac{1}{n}\right)\log|q_n\alpha-p_n|=\frac{\pi^2}{12\log 2}$.

The purpose of this paper is to give an extension of above theorems to inhomogeneous linear forms $\alpha x+\beta-y$. Morimoto ([4]) presented a generalized algorithm of the simple continued fraction expansion, which induces vertex points (q_n, p_n)

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