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On an Asymptotic Property of a Nonlinear Ordinary Differential Equation

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§1. Introduction.

In the study of the fifth Painlevé equation we treated an equation of the form

$$x(xu')'=\frac{\alpha}{2}\tanh u\cosh^{-2}u+\frac{\gamma}{4}x\sinh 2u+\frac{\delta}{8}x^2\sinh 4u \tag{1}$$

($'=d/dx$), where $\alpha, \gamma\in \boldsymbol{R}$, $\delta<0$. In [4] we studied an asymptotic behaviour of the solution $u=u_0(x)=u(x_0, u_0, u'_0; x)$ ($x_0>0$, $u_0, u'_0\in \boldsymbol{R}$) as $x\to+\infty$ satisfying an initial condition

$$u_0(x_0)=u_0\,, \qquad u'_0(x_0)=u'_0\,. \tag{2}$$

In this paper we consider a more general nonlinear equation of the form

$$v''+v\Phi(x, v)=0\,. \tag{3}$$

Under some assumptions we prove that the solution $v=V(x)$ satisfying an initial condition as above can be prolonged over the interval $x_0\le x<+\infty$, and we give an asymptotic expression of $V(x)$ as $x\to+\infty$. Analogous problems are studied in [1], [2] and [3].

§2. Main result.

Let r and ε be positive constants. Consider an equation of the form

$$u''+u(1+x^{-1}p(u)+x^{-1-\varepsilon}f(x, u))=0 \tag{4}$$

satisfying the following conditions.

(A) *$p(u)$ is a polynomial of degree $2n$ (≥ 0)*

$$p(u)=\lambda_0+\lambda_1u+\cdots+\lambda_{2n}u^{2n}$$

where

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