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A Note on the Rational Approximations to *e*

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Introduction.

We have some consequences on the rational approximations to e. P. Bundschuh [2] proved the following inequality.

BUNDSCHUH'S THEOREM. *For all integers p, q such that $q>0$,*

$$\left|e-\frac{p}{q}\right|>\frac{\log\log 4q}{18q^2\log 4q}.$$

On the other hand, C. S. Davis [3] proved the following theorem.

DAVIS' THEOREM. *For any $\varepsilon>0$ there is an infinity of solutions of the inequality*

$$\left|e-\frac{p}{q}\right|<\left(\frac{1}{2}+\varepsilon\right)\frac{\log\log q}{q^2\log q}$$

in integers p, q. Further, there exists a number $q'=q'(\varepsilon)$ such that

$$\left|e-\frac{p}{q}\right|>\left(\frac{1}{2}-\varepsilon\right)\frac{\log\log q}{q^2\log q}$$

for all integers p, q with $q\geqq q'$.

The last inequality suggests the possibility of replacing the constant 1/18 in Bundschuh's theorem by a larger one; which will be done in this note.

THEOREM. *Let p, q be positive integers such that $q\geqq 2$. Then*

$$\left|e-\frac{p}{q}\right|>\frac{\log\log q}{3q^2\log q}.$$

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