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The Involutions of Compact Symmetric Spaces, II

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Dedicated to Professor Ichiro Satake on his Sixtieth Birthday

Introduction. This is a part of our series on a geometric theory of the compact symmetric spaces M ([CN-1], [CN-2], [CN-3], [NS-1], [NS-2], [N]); especially we will give proofs to certain statements in [N] (as was promised there). The theory features the subspaces, called the polars and the meridians (1.5 for both) of M, which are significant building blocks of maximal size (i.e. larger than the cells in any reasonable decomposition such as that of Bruhat). The polars are directly related to the space M in topology; 1.11 is just one example. They are the critical submanifolds of a certain Bott-Morse function (at least if M is an R-space; see [T-1]), and so forth. More strikingly, even the signature (or index) of M equals the sum of that of the polars, of which we do not have a direct proof yet. We point out another intriguing fact about polars; Uhlenbeck [U] found that every harmonic map from the 2-sphere into $U(n)$ is a product of those into polars. On the other hand, the meridians have equal rank to that of M and their root systems are related to that of M with a simple rule (2.15); therefore their curvature is related to that of M in an equally simple way. The polars are paired with the meridians; they are the "orthogonal complements" to each other, while the polars are the connected components of the fixed points of any one of the involutions by which M becomes a symmetric space (1.5). Their theoretical significance lies in the fact that M is determined by a single pair of a polar and a meridian (1.15). Thus it is an easy corollary that a simple M is hermitian if and only if a polar and a meridian in a pair are hermitian (2.30). Also the theory aims at studies of interrelationship between symmetric spaces or morphisms $f: B \to M$ between them. The case of $\dim B = 0$ was studied in [CN-3] and [T-3]. The case of B = sphere was done in [NS-1] fairly completely.

In section 1, we will explain basic concepts and facts about them as well as their relevance in our geometric theory. Theorem 1.8 gives the basic property of the meridian in connection with the maximal tori, whose proof includes a new proof of conjugacy

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