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On Wall Manifolds with Almost Free Z_{2^k} Actions

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0. Introduction.

In order to understand the bordism classification of finite group actions on oriented manifolds, it is useful to consider some notion of manifolds with equivariant Wall structures. In [8], C. Kosniowski and E. Ossa studied the bordism theory $W_*(Z_2; All)$ of Wall manifolds with unrestricted involutions and determined completely the bordism theory $\Omega_*(Z_2; All)$ of oriented involutions, especially its torsion part as the image of the Bockstein homomorphism $\beta: W_*(Z_2; All) \to \Omega_*(Z_2; All)$. In this paper, we treat an almost free Z_{2^k} action on Wall manifold, i.e., one for which only the $Z_2 \subset Z_{2^k}$ may possibly fix points on manifold. From the viewpoint of action, such object is exactly Wall manifold with action of type $(Z_{2^k}, 1)$ in [13].

In section 1, we study the bordism theory $W_*(Z_{2^k}; Af)$ of these objects. By the map which ignores Wall structures, the theories $W_*(Z_{2^k}; Free)$ and $W_*(Z_{2^k}; Af, Free)$ are derived from the corresponding unoriented theories as usual (Propositions 1.4 and 1.8). In particular, we have that $W_*(Z_{2^k}; Af, Free)$ is the sum of three parts; the images $Im(t)$ of two kinds of extensions from Z_2 actions and another part $\bar{L}_*$. Using these results, we obtain the exact sequence for the triple $(Af, Free, \emptyset)$ (Proposition 1.11), and the W_*-module structure of $W_*(Z_{2^k}; Af)$ (Theorem 1.19). There the classes $\{V(0, 2n+2)\}$ (Definition 1.17) are useful to describe the part K_t which lies in $Im(t) \subset W_*(Z_{2^k}; Af, Free)$, while the part L_* is isomorphic to $\bar{L}_*$ naturally.

In section 2, we describe the image $\mathscr{T}$ of the map β: $W_*(Z_{2^k}; Af) \to \Omega_*(Z_{2^k}; Af)$; the bordism module of orientation preserving almost free Z_{2^k} actions, and describe the torsion part of order 2 (Theorem 2.3). As an application, we study the image of $I_*: \Omega_*(Z_4; Free) \to \Omega_*(Z_4; Af)$; the forgetful homomorphism by using the result of principal Z_{2^k} actions in [5] (Theorem 2.9).

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