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# Classifying Hypersurfaces in the Lorentz-Minkowski Space with a Characteristic Eigenvector

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## 1. Introduction.

In a famous paper, [3], Cheng and Yau solved the Bernstein problem in the Lorentz-Minkowski space $L^{n+1}$ showing that the only entire maximal hypersurfaces are hyperplanes. Maximal and constant mean curvature (CMC) hypersurfaces play a chief role in relativity theory as it is pointed out in a series of papers by Choquet, Fischer and Marsden, [4], Stumbles, [15], and Marsden and Tipler, [13]. CMC hypersurfaces are often closely related to either an eigenvalue problem or a differential equation stemming from the Laplacian. Perhaps the most remarkable case is that concerning to vanishing constant mean curvature. Let $x$ denote an isometric immersion of a hypersurface $M$ in the Lorentz-Minkowski space $L^{n+1}$ and let $H$ be the mean curvature vector field. In a recent paper, Markvorsen, [12], gives a pseudo-Riemannian version of the well-known Takahashi's theorem showing that the coordinate functions of the immersion $x$ are eigenfunctions of the Laplacian $\Delta$ of $M$, associated to the same eigenvalue $\lambda$, if and only if $M$ is a vanishing CMC hypersurface ($\lambda=0$), a de Sitter space $S_1^n(r)$ ($\lambda>0$) or a hyperbolic space $H^n(r)$ ($\lambda<0$). That means that vanishing mean curvature hypersurfaces in $L^{n+1}$ are the only ones having harmonic coordinate functions.

More recently, Garay and Romero, [8], ask for hypersurfaces in $L^{n+1}$ satisfying the condition $\Delta H=C$, $C$ being a constant vector of $L^{n+1}$ which is normal to $M$ at every point, and show that $C$ should vanish. As for surfaces in $L^3$, we have shown in [7] that vanishing mean curvature surfaces are the only ones satisfying $\Delta H=0$, so that it seems natural to ask for the following geometric question:

> Does the equation $\Delta H=0$ characterize the vanishing CMC hypersurfaces of $L^{n+1}$?

That equation motivates ourselves to study a certain generalization of Takahashi's

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