On a Certain Class of Generalized Hypergeometric Functions with Finite Monodromy Groups *)

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Introduction.

In this paper we shall investigate the monodromy group G of generalized hypergeometric equation (say GHGE, for brevity) in the form of Okubo type, its irreducibility conditions, explicit form of its invariant hermitian matrix and so on. As we shall see later, there exists a complex reflection group $\tilde{G}$ induced from G and containing G. We shall also study it and determine the case where $\tilde{G}$ is a finite irreducible group. Then G is also finite and the corresponding solutions of GHGE are algebraic functions.

The equation of Okubo type is a system of first order linear differential equations (see (#) in §1). It gives a fine perspective in the theory of Fuchsian equations to consider those in the form of Okubo type. K. Okubo showed in [5] that every Fuchsian equation can be transformed into one of his type and its monodromy group up to conjugations can be obtained by an algebraic computation. In particular, for GHGE, he obtained the monodromy group in the joint work with Takano ([6]) and solved the connection problem ([5], see also [7]). His theory says that n-th order Fuchsian equations in general have $n^2 - 3n + 2$ numbers of accessory parameters (see [5]). We may comprehend above $\tilde{G}$ as the monodromy group of such equation, of which the number of accessory parameters takes the special value 0, and GHGE as a limit of those equations.

For the single higher order GHGE, the monodromy group was obtained by A. H. M. Levelt in his thesis [4]. We may say that it is *integral*, i.e., it is a subgroup of $GL(n, \mathcal{O}_K)$ if its parameters are rational numbers, where $\mathcal{O}_K$ is the ring of integers of a suitable cyclotomic field K. Recently, in their joint work ([1]), F. Beukers and G. Heckman investigated systematically the cases where the group due to Levelt come to be finite. Theorem 4.8 in [1] has a particular importance which was brought by virtue of the above integral property besides their good idea. Moreover they obtained various consequences by studying a reflection subgroup of the monodromy group. The fact is