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On the Galois Group of $x^p+p^tb(x+1)=0$

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1. In [3] we discussed the Galois group of

$$x^p+ax+a=0$$

over the rational number field $\boldsymbol{Q}$, where p is a prime number, and $a\in\boldsymbol{Z}$, $(p, a)=1$. The situation becomes much more complicated when a is divisible by p. In this paper we deal with three special cases:

1. $a=p^tb$, $0<t<p$, $(p, b)=1$, $|(p-1)^{p-1}b+p^{p-t}|$ is not a square;
2. $a=pk^2$, $(p, k)=1$;
3. $a=p^{2m}b$, $0<2m<p$, $(p, b)=1$.

We begin by proving the following theorem (cf. [3]).

THEOREM 1. *Let $a_0, a_1, \cdots, a_{n-1}$ be rational integers such that*

$$f(x)=x^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$$

is irreducible over the rational number field $\boldsymbol{Q}$. Let α be a root of $f(x)=0$, and let

$$\delta=f'(\alpha)\,,\qquad D=\operatorname{norm}\delta\ (in\ \boldsymbol{Q}(\alpha))\,,$$

$$D/\delta=x_0+x_1\alpha+\cdots+x_{n-1}\alpha^{n-1}\,,\qquad x_i\in\boldsymbol{Z}\,.$$

Let D_1 and D_2 denote any rational integers which satisfy the following conditions:

$$D=D_1D_2\,,\tag{1.1}$$

$$(D_1, D_2)=1\,,\tag{1.2}$$

$$(D_2, x_0, x_1, \cdots, x_{n-1})=1\,.\tag{1.3}$$

Let G denote the Galois group of $f(x)=0$ over $\boldsymbol{Q}$; G is a transitive permutation group on the set $\{1, 2, \cdots, n\}$. Then we have:

I. *If $|D_2|$ is not a square, G contains a transposition.*
II. *If $|D_2|$ is a square, D_1 is divisible by the discriminant of $\boldsymbol{Q}(\alpha)$.*

PROOF. Suppose first that $|D_2|$ is not a square. Then there exists a prime number

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