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Symplectic Manifolds with Semi-Free Hamiltonian S^1-Action

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1. Introduction

In a previous paper [H1] we have shown that a connected closed symplectic S^1-manifold with the properties (i), (ii) and (iii) listed below must be simply connected and have the same homology groups as $S^2 \times \cdots \times S^2$.

(i) The action admits a moment map.

(ii) The fixed points are all isolated.

(iii) The action is semi-free.

In this paper we shall prove the following

THEOREM 1.1. *Let M be a connected closed symplectic S^1-manifold satisfying the conditions* (i), (ii) *and* (iii) *listed above. Then M has the same cohomology ring and the same Chern classes as $S^2 \times \cdots \times S^2$.*

Semi-free actions are the simplest and, in a sense, the basic type among S^1-actions. For example, if M is any S^1-manifold and if $\boldsymbol{Z}/m$ is a maximal finite isotropy subgroup then each component of the fixed point set of the restricted $\boldsymbol{Z}/m$-action is an invariant symplectic S^1-submanifold on which the S^1-action, made effective, is free or semi-free. Thus our result above may be considered as the first step to investigate general symplectic S^1-manifolds admitting moment map.

The main idea of proof can be stated as follows. The critical points of the moment map are precisely the fixed points of the action. Let $\Sigma_1, \cdots, \Sigma_n$ be a suitably chosen homology basis of $H_2(M)$ corresponding to the fixed points of index 2 and let $x_1, \cdots, x_n$ be the dual basis of $H^2(M)$. Let $\xi=\xi(h_1, \cdots, h_n)$ be the complex line bundle over M with $c_1(\xi)=x=h_1x_1+\cdots+h_nx_n$ where $h_i \in \boldsymbol{Z}$. The S^1-action on M can be lifted to an action on ξ and defines a weight $a_i=a_i(h_1, \cdots, h_n)$ at each fixed point P_i. These weights a_i satisfy certain relations coming from a fixed point formula. Also they are related to $x^n[M]$. Using these two facts and the linearity of x and a_i with respect to $h_1, \cdots, h_n$ we determine all the values $x_{i_1}x_{i_2}\cdots x_{i_n}[M]$ which, in turn,

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