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On the Iwasawa λ-Invariants of Real Quadratic Fields

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Introduction.

Let k be a finite extension of the field of rational numbers $\boldsymbol{Q}$ and p a fixed prime number. A Galois extension K of k is called a $\boldsymbol{Z}_p$-extension if the Galois group $\mathrm{Gal}(K/k)$ is topologically isomorphic to the additive group $\boldsymbol{Z}_p$ of the p-adic integers. Every number field k has at least one $\boldsymbol{Z}_p$-extension, namely the cyclotomic $\boldsymbol{Z}_p$-extension which is contained in the field obtained by adjoining all p-power roots of unity to k.

For a $\boldsymbol{Z}_p$-extension

$$k=k_0\subset k_1\subset k_2\subset\cdots\subset k_n\subset\cdots\subset K=\bigcup_{n=1}^{\infty}k_n$$

with Galois groups $\mathrm{Gal}(k_n/k)\simeq\boldsymbol{Z}/p^n\boldsymbol{Z}$, let h_n be the class number of k_n and p^{e_n} the exact power of p dividing h_n. Then Iwasawa has proved that there exist integers λ, μ and ν, depending only on K/k and p, such that $e_n=\lambda n+\mu p^n+\nu$ for all sufficiently large n. The integers $\lambda=\lambda_p(K/k)$, $\mu=\mu_p(K/k)$ and $\nu=\nu_p(K/k)$ are called the Iwasawa invariants of K/k for p. For convenience, the Iwasawa invariants of the cyclotomic $\boldsymbol{Z}_p$-extension of k for p will be denoted by $\lambda_p(k)$, $\mu_p(k)$ and $\nu_p(k)$.

In [6], Greenberg stated the following conjecture concerning the Iwasawa invariants:

"*If k is totally real, then both $\lambda_p(k)$ and $\mu_p(k)$ vanish.*"

It seems quite difficult to decide whether this conjecture is true, even for real quadratic fields.

Recently in [2], [3], [4] and [5], Fukuda and Komatsu studied Greenberg's conjecture in some real quadratic cases. They defined two invariants n_1 and n_2 in [4] (cf. Section 1), and treated the cases where $2\leq n_1<n_2$ and $n_1=1$ in [3], [4] and the case where $n_1=n_2=2$ in [2], [5] (See Addendum).

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