TOKYO J. MATH.
VOL. 16, NO. 1, 1993

Logarithmic Projective Connections

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In this note, we consider logarithmic version of the results in [K3]. Let X be a complex manifold of dimension $n\geq 2$ and D a reduced effective divisor on X with only normal crossing singularities. We define logarithmic projective Weyl forms $\bar{P}_k$, $0\leq k\leq n$, a kind of characteristic forms of the pair (X, D), by means of a C^∞-logarithmic projective connection and prove the following formula

$$\bar{c}_k(\bar{\theta})=\sum_{j=0}^{k}\binom{n+1-j}{k-j}((n+1)^{-1}\bar{c}_1(\bar{\theta}))^{k-j}\bar{P}_j(\bar{\pi}), \qquad 0\leq k\leq n,$$

where the $\bar{c}_k(\bar{\theta})$ are the logarithmic Chern forms defined by a suitable C^∞-logarithmic affine connection $\bar{\theta}$ (Theorem 3.1). If X is compact, Kähler and admits a holomorphic logarithmic projective connection, then the logarithmic projective Weyl forms are d-exact. Hence, in this case, our formula gives the formula on the logarithmic Chern classes

$$\bar{c}_k=\binom{n+1}{k}((n+1)^{-1}\bar{c}_1)^k, \qquad 1\leq k\leq n.$$

The latter is the logarithmic version of Gunning's formula [G], see also [K3].

In the last section, we shall reprove a formula on Chern classes of certain compact non-Kähler 3-folds which were constructed in [K1] as an application of our main result.

NOTATION.

Ω^p : the sheaf of germs of holomorphic p-forms on a complex manifold,

$\Omega^p(\log D)$: the sheaf of germs of logarithmic p-forms along a divisor D on a complex manifold,

$\mathcal{O}\simeq\Omega^0$: the sheaf of germs of holomorphic functions on a complex manifold,

Θ : the sheaf of germs of holomorphic vector fields on a complex manifold,

$\Theta(-\log D)$: the sheaf of germs of logarithmic vector fields along a divisor D on a complex manifold,

$\mathscr{A}^r(E)$: the sheaf of germs of differentiable r-form-valued sections of a vector

Received February 10, 1992