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Foliations on Manifolds with Positive Constant Curvature

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§1. Introduction.

Geometric notions in the theory of Riemannian submanifolds have their counterparts for foliations on Riemannian manifolds. A foliation all of whose leaves are minimal (resp. totally geodesic) submanifolds is called a harmonic (resp. totally geodesic) foliation and has been extensively studied in recent years. Many harmonic foliations which are not totally geodesic are known. However, under some geometric restrictions, harmonicity implies totally-geodesicness. For example, Oshikiri [13] proves that any harmonic foliation of codimension 1 on a compact orientable Riemannian manifold with non-negative Ricci curvature is totally geodesic. This result fails to hold in the case of higher codimensions (see Takagi and Yorozu [15]). But for harmonic foliations on a space-form, the following conjecture has been known:

CONJECTURE 1. Any harmonic foliation with minimal normal plane field on a compact n-dimensional space-form $M^n(c)$ of constant sectional curvature c ($c \geq 0$) is totally geodesic.

As an analogue when the ambient space is a complex space-form, the following is also conjectured:

CONJECTURE 2. Any harmonic foliation with minimal normal plane field on the complex projective space $P_n(C)$ with its standard metric is totally geodesic.

It should be mentioned that Escobales [5] proves that any totally geodesic foliation with bundle-like metric on a rank one symmetric space consists of fibres of a Hopf fibration. Nakagawa and Takagi [11] claimed that Conjecture 1 was true and Gotoh [6] successively gave a proof of Conjecture 2 by using similar technique. However, Li [10] pointed out an error in their proof and these conjectures still remain unsolved.

In this paper, we shall obtain partial affirmative answers to these conjectures.

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