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Exponential Kummer Coverings and Determinants of Hypergeometric Functions

Tomohide TERASOMA

Tokyo Metropolitan University

§1. Introduction.

In the papers of Aomoto [A1][A2], he discovered a generalization of hypergeometric function of Appell's hypergeometric function and studied the monodromy of the differential equation defined by this Aomoto-Gel'fand hypergeometric function. This generalized hypergeometric function is defined as an integral of differential form on some topological cycle. Recently, this integral is known to be closely related to a period analogue of l-adic representation of profinite braid group or generalized braid group. The explicit formula for the determinant of arithmetic Magnus representation is given in [O-T]. In this paper we treat the period analog of the above paper.

We explain the results of this paper. Let n be an integer such that $n\geq 3$, $\lambda_1, \cdots, \lambda_n$ and $\alpha_1, \cdots, \alpha_n$ be real numbers such that $\lambda_1<\cdots<\lambda_n$ and $\alpha_i>0$ respectively. Let a_{ij} $(1\leq i, j\leq n-1)$ be a singular integral of Jordan-Pochhammer type defined by

$$a_{ij}=\int_{\lambda_i}^{\lambda_{i+1}}\prod_{p=1}^{i}(x-\lambda_p)^{\alpha_p-1}\prod_{p=i+1}^{n}(\lambda_p-x)^{\alpha_p-1}x^{j-1}dx\,.$$

THEOREM 1. *The determinant of* $A=(a_{ij})_{1\leq i,j\leq n-1}$ *is given by*

$$\det A=\prod_{i=1}^{n}\{(-1)^{i-1}\prod_{j\neq i}(\lambda_j-\lambda_i)\}^{\alpha_i}\prod_{1\leq i<j\leq n}(\lambda_j-\lambda_i)^{-1}\cdot\frac{\Gamma(\alpha_1)\cdots\Gamma(\alpha_n)}{\Gamma(\alpha_1+\cdots+\alpha_n)}\,.$$

This theorem is proved by Varchenko [Var]. In this paper, we give another direct proof of this determinant theorem. For the intermediate exterior product for Appell's hypergeometric functions, we have the following theorem.

THEOREM 2. *For an integer r such that* $1\leq r\leq n-1$, *and sets of indices*

$$I\in\{(i_1, \cdots, i_r)\mid 0\leq i_1<\cdots<i_r\leq n-2\}\,,\quad J\in\{(j_1, \cdots, j_r)\mid 0\leq j_1<\cdots<j_r\leq n-2\}\,,$$

we define $A_{I,J}$ *as the* (I, J)*-minor of the matrix* $(a_{i,j})_{i\in I, j\in J}$ *defined as above. Let* $\bar{\Omega}_I$ *be a differential form;*

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