TOKYO J. MATH.
VOL. 16, NO. 2, 1993

Positive Characteristic Finite Generation of Symbolic Rees Algebras and Roberts' Counterexamples to the Fourteenth Problem of Hilbert

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1. Introduction.

In this paper we shall study the problem whether the symbolic Rees algebras occurring in Roberts' new counterexamples [10] to the 14th problem of Hilbert are Noetherian rings or not, in the case where the characteristic of the ground field is positive.

For each prime ideal Q in a commutative Noetherian ring R we put $R_s(Q)=\sum_{n\geq 0}Q^{(n)}\xi^n$ and call it the *symbolic Rees algebra* of Q, where $Q^{(n)}$ denotes the n^{th} symbolic power of Q and ξ is an indeterminate over R. The determination of finite generation in $R_s(Q)$ is one of the central problems in both commutative algebra and algebraic geometry (cf. [8], [7], [9], [10], [3], and [4]). It is generally a quite hard problem but, according to the recent research [4], in the positive characteristic case there might be more chances for $R_s(Q)$ to be a Noetherian ring than in the case where the characteristic is zero.

Originally this kind of question was raised in 1985 by Cowsik [3], asking if $R_s(Q)$ are always Noetherian especially when the base ring R is regular (and local). However, as is now well known, this is not true in general. Three counterexamples [9], [10], and [4] are already known. In this paper we are particularly interested in the second example [10] due to Roberts, so we would like to cite here his examples explicitly.

Let F be a field and $R_0=F[x, y, z]$ be a polynomial ring with three indeterminates over F. $R=R_0[S, T, U, V]$ and $R_0[W]$ denote polynomial rings over R_0. For each positive integer t let $\varphi: R\to R_0[W]$ be the homomorphism of R_0-algebras defined by $\varphi(S)=x^{t+1}W$, $\varphi(T)=y^{t+1}W$, $\varphi(U)=z^{t+1}W$ and $\varphi(V)=(xyz)^tW$. We put $Q=\mathrm{Ker}(\varphi)$. Let $R_1=R_0\cdot S+R_0\cdot T+R_0\cdot U+R_0\cdot V$ be a free R_0-module and let $\phi: R_1\to R_0$ be an R_0-linear map such that $\phi(S)=x^{t+1}$, $\phi(T)=y^{t+1}$, $\phi(U)=z^{t+1}$ and $\phi(V)=(xyz)^t$. We denote by M the kernel of ϕ and put $S(M)=R_0[M]$ ($\subseteq R$). Let $\overline{S(M)}$ be the ideal-transform of

Received September 14, 1992
Revised February 26, 1993
Partially supported by Grant-in-Aid for Co-operation Research