

On the Intermittency of a Piecewise Linear Map (Takahashi Model)

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1. Introduction.

We will consider a piecewise linear mapping F from the unit interval $[0, 1]$ into itself for which there exists a sequence $\{c_n\}$ such that $1 = c_0 > c_1 > \cdots \rightarrow 0$ and

$$F(x) = \begin{cases} \frac{x - c_1}{\psi_0} & \text{if } x \in (c_1, c_0], \\ \frac{x - c_{n+1}}{\psi_n} + c_n & \text{if } x \in (c_{n+1}, c_n], \end{cases}$$

where

$$\begin{aligned} \psi_n &= F'(x)^{-1} && \text{if } x \in (c_{n+1}, c_n) \\ &= \begin{cases} \frac{c_0 - c_1}{c_0} & \text{if } n = 0, \\ \frac{c_n - c_{n+1}}{c_{n-1} - c_n} & \text{if } n \geq 1. \end{cases} \end{aligned}$$

Y. Takahashi ([7] and [8]) studied the ergodic properties of this mapping by calculating its autocorrelations $\int xF(x)dx$. In this paper, we will consider this problem in more general situation.

This mapping has a fixed point at 0, and there are various cases depending on its property at 0:

1. When $F'(0) > 1$, the dynamical system is mixing and the decay rate of correlation is of exponential order. In particular, the central limit theorem holds.
2. When $F(x) - x \sim x^\alpha$ such that $\alpha < 5/3$, the dynamical system is still mixing, but the decay rate of correlation is of polynomial order. Nevertheless, the central limit