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Explicit Reciprocity Formulas in 2-Adic Number Fields

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Dedicated to Professor Katsumi Shiratani on his 60th birthday

§1. Introduction.

Let $k/\boldsymbol{Q}_2$ be a finite extension, where $\boldsymbol{Q}_2$ denotes the 2-adic rational number field. In the present paper, we give some explicit formulas for the generalized Hilbert norm residue symbol on the field generated by prime-power division points of a Lubin-Tate formal group defined over k.

Our formulas are 2-adic versions of Shiratani's explicit formulas [7] which generalize Takagi's formulas [9] for the prime cyclotomic field. These formulas give explicit reciprocity laws for certain generators of the multiplicative group and of the formal module, and consist of *complementary laws* and *general laws*.

In §3, we give a complementary law which is slightly different from the odd p case. In §4, we give a general law which has the same shape as the odd p case. For odd p, formulas of this type can also be obtained by a different method [8].

§2. Preliminaries.

Let $\mathfrak{o}$ be the integer ring of k and $\mathfrak{p}$ the prime ideal of $\mathfrak{o}$. For a prime element π of k, let $F(X, Y)\in\mathfrak{o}[[X, Y]]$ be a Lubin-Tate formal group belonging to π. This implies that F is a one-dimensional commutative formal $\mathfrak{o}$-group law such that the endomorphism $[\pi]_F$ associated with π satisfies

$$\begin{cases} [\pi]_F(X)\equiv X^q & \mod \pi\,, \\ [\pi]_F(X)\equiv \pi X & \mod \deg 2\,, \end{cases}$$

where $q=2^f$ is the number of elements in the residue field $\mathfrak{o}/\mathfrak{p}$. Let v_n be a primitive π^n-division point of F in the algebraic closure k_s of k such that $[\pi]_F(v_n)=v_{n-1}$ for $n\geq 2$. Let $k_n=k(v_n)$. We denote by $\mathfrak{p}_n$ the prime ideal of k_n and by $F(\mathfrak{p}_n)$ the associated formal module. For $\alpha\in k_n^\times$ and $\beta\in F(\mathfrak{p}_n)$, the generalized Hilbert symbol $(\alpha, \beta)_n^F$ is defined [10] by

$$(\alpha, \beta)_n^F=\rho^{\sigma_\alpha}\underset{F}{-}\rho\,, \qquad \rho\in k_s\,, \quad [\pi^n]_F(\rho)=\beta\,,$$

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