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On the Topology of Fermat Type Surface of Degree 5 and the Numerical Analysis of Algebraic Curves

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1. Introduction.

Let V_n be a Fermat type algebraic surface of degree n, that is,

$$V_n=\{[z_0:z_1:z_2:z_3]\in \boldsymbol{CP}^3 \mid z_0^n-z_1^n-z_2^n+z_3^n=0\} .$$

We consider a fibration $f: V_n \to \boldsymbol{CP}^1$ given by

$$f:[z_0:z_1:z_2:z_3]\mapsto\begin{cases}[z_2^{n-1}:z_0^{n-1}] & \text{if } z_0=z_1 \text{ and } z_2=z_3\\ [z_0-z_1:z_2-z_3] & \text{otherwise}.\end{cases}$$

A general fiber of f is a Riemannian surface of genus $(n-2)(n-3)/2$. If $n\leq 4$, a general fiber is a sphere or a torus and the singular fibers and their monodromies are known. (See [K].). In the case $n=5$ the genus of a general fiber is 3. Matsumoto calculates in his notes [M] the positions and homeomorphism-types of all singular fibers appearing in the fibration $f: V_n \to \boldsymbol{CP}^1$ for general n. From his results we know the conjugate class of the local monodromy for each singular fiber.

In this paper we suppose $n=5$ and we give an algorithm to calculate the global monodromy map

$$[\tilde{\rho}_*]: \pi_1(\boldsymbol{CP}^1-SF, \sigma_0) \to \mathfrak{M}_3=\mathrm{Aut}\,\pi_1\Sigma_3/\mathrm{Inn}\,\pi_1\Sigma_3$$

using numerical analysis of algebraic curves in $\boldsymbol{CP}^2$. Here

$$SF=\{\sigma \mid F_\sigma=f^{-1}(\sigma) \text{ is a singular fiber}\} .$$

First we define a branched covering map

$$h_\sigma: F_\sigma \to \boldsymbol{CP}^1$$

for each general fiber $F_\sigma=f^{-1}(\sigma)$. Its branch loci are obtained as solutions of certain

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