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0-Dimensional Moduli Space of Stable Rank 2 Bundles and Differentiable Structures on Regular Elliptic Surfaces

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1. Introduction.

The study of moduli spaces of stable rank 2 vector bundles over smooth complex algebraic surfaces has contributed to the study of differentiable structures on their underlying smooth 4-manifolds through Donaldson's result in [Do1]: the moduli space of irreducible Hermitian-Einstein connections on a $U(2)$-bundle P coincides with the moduli space of stable holomorphic structures on P. By Friedman, Morgan, Okonek, Van de Ven and others, a lot of work on moduli spaces of stable bundles over certain algebraic surfaces (for example, Dolgachev surfaces) has been done ([Fri], [FrM1], [LO], [OV]), and they showed that their underlying topological 4-manifolds admit distinct differentiable structures. The bundles which they considered have trivial first Chern classes and moduli spaces of nonzero dimension. In [Kot] Kotschick studied the 0-dimensional moduli space of stable bundles $\mathscr{E}$ with $c_1(\mathscr{E})=K_B$ and $c_2(\mathscr{E})=1$ over the Barlow surface B, and he showed that the Barlow surface B is homeomorphic but not diffeomorphic to $CP^2 \# 8\overline{CP^2}$. Here, K_B is the canonical divisor of B.

In this paper, we consider 0-dimensional moduli spaces of stable bundles with nontrivial first Chern classes. Our target surfaces are certain generic regular elliptic surfaces S, that is, S is a compact complex surface with a certain type of holomorphic map $\pi: S \to CP^1$ called an elliptic fibration and with a section. We show that for a certain ample divisor L, the moduli space of L-stable rank 2 bundles over S with $c_1=\Sigma-K_S$ and $c_2=1$ consists of exactly one point. Here Σ is a section of π. Moreover, by using a result about the effect of surgery on the simple invariants obtained by Gompf and Mrowka [GoM], we show the following, which was announced by Friedman and Morgan in [FrM2]:

THEOREM. *Let S be a minimal elliptic surface with strictly positive geometric genus. Then the underlying topological 4-manifold of S admits infinitely many differentiable structures.*

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