

On Connectedness of the Space of Harmonic 2-Spheres in Quaternionic Projective Spaces

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(Communicated by T. Nagano)

Introduction.

The research on the spaces of harmonic maps of compact Riemann surfaces is one of important areas in harmonic map theory. Recently the connectedness of the spaces of harmonic 2-spheres in specific Riemannian manifolds has been investigated by several mathematicians. Kotani [Ko] showed that the space of harmonic 2-spheres in the n -dimensional standard sphere S^n with fixed energy is path-connected if $n \geq 3$. In case $n=4$, this result was previously proved by Loo [Lo] and Verdier [Ve]. Furthermore Guest and Ohnita [GO] investigated group actions on harmonic maps into symmetric spaces and used Morse-Bott theoretic deformations for harmonic maps to show some results on the connectedness of the space of harmonic 2-spheres in the unitary group, the sphere and the complex projective space. Moreover the fundamental group of the space of harmonic 2-spheres in the n -sphere was determined by [FGKO]. It is natural to study the connectedness of the space of harmonic 2-spheres in the quaternionic projective space.

Let HP^n be an n -dimensional quaternionic projective space. It is known that there are two natural twistor spaces \mathcal{T}_n and CP^{2n+1} over HP^n (see Section 1). A harmonic map $\varphi: \Sigma \rightarrow HP^n$ is *strongly isotropic* if and only if φ can be lifted to a horizontal holomorphic map into \mathcal{T}_n (see [G1]). According to [BED-W], $\varphi: \Sigma \rightarrow HP^n$ is called a *quaternionic mixed pair* if φ can be lifted to a horizontal holomorphic map into CP^{2n+1} .

Denote by c the maximum of the sectional curvatures of HP^n . Let $\varphi: \Sigma \rightarrow HP^n(c)$ be a harmonic map of a compact Riemann surface. If φ is strongly isotropic or a quaternionic mixed pair, then φ has energy $4\pi d/c$, for some nonnegative integer d (see Section 2).

The purpose of this paper is to prove the following theorem, by virtue of the method of [GO] applied to the twistor spaces \mathcal{T}_n and CP^{2n+1} .

THEOREM A. *The space of harmonic 2-spheres in $HP^n(c)$ with fixed energy $4\pi d/c$*