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Some Remarks on the Characterization of the Poisson Kernels for the Hyperbolic Spaces

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Introduction.

Let G be a classical connected simple Lie group of real rank 1: i.e. G is one of the groups $SO_0(1, n)$, $SU(1, n)$ and $Sp(1, n)$ corresponding to the fields $\boldsymbol{R}$, $\boldsymbol{C}$ and $\boldsymbol{H}$ respectively. Let $G=KAN$ be an Iwasawa decompositon and M be the centralizer of A in K. Denoting by $\boldsymbol{F}$ the field corresponding to the group G, then G/K is the classical hyperbolic space, i.e. the unit ball in $\boldsymbol{F}^n$ (denoted by $B(\boldsymbol{F}^n)$) and it's Martin boundary K/M is the unit sphere in $\boldsymbol{F}^n$ (denoted by $S(\boldsymbol{F}^n)$). The action of G on $B(\boldsymbol{F}^n)$ and $S(\boldsymbol{F}^n)$ is concretely given as follows: for $x={}^t(x_1, \cdots, x_n)\in \boldsymbol{F}^n$ and $g=(g_{pq})_{0\leq p,q\leq n}\in G$, we define

$$x'=gx\,,$$

where $x'={}^t(x'_1, \cdots, x'_n)$, with

$$x'_p=(g_{p0}+\sum_{q=1}^{n} g_{pq}x_q)(g_{00}+\sum_{q=1}^{n} g_{0q}x_q)^{-1}\,, \qquad 1\leq p\leq n\,.$$

And the identifications $G/K\cong B(\boldsymbol{F}^n)$ and $K/M\cong S(\boldsymbol{F}^n)$ are given by

$$G/K\cong B(\boldsymbol{F}^n)\ ;\ gK\mapsto gO\,,$$

$$K/M\cong S(\boldsymbol{F}^n)\ ;\ kM\mapsto ke_1\,,$$

where O is the origin of $\boldsymbol{F}^n$ and $e_1={}^t(1, 0, \cdots 0)\in S(\boldsymbol{F}^n)$.

We now denote by D the Laplace-Beltrami operater on $G/K\cong B(\boldsymbol{F}^n)$. The Poisson kernel P: $G/K\times K/M\to \boldsymbol{R}$ is given as follows:

$$P(gK, kM)=\left(\frac{1-|x|^2}{|1-{}^t\bar{x}\cdot b|^2}\right)^{\rho}\,,$$

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