TOKYO J. MATH.
VOL. 17, NO. 1, 1994

On the First Coefficients in q of the Kazhdan-Lusztig Polynomials

Hiroyuki TAGAWA

University of Tokyo

(Communicated by T. Nagano)

§0. Introduction.

The purpose of this article is to find a combinatorial description of the first coefficient in q of the Kazhdan-Lusztig polynomial (Theorem A) by introducing a left subword, which is a special one of subwords (see Def. 1.6). From its description, we show the non negativity of the first coefficient in q of the Kazhdan-Lusztig polynomial for x, w satisfying $l(w)=l(x)+l(x^{-1}w)$ or $l(w)=l(x)+l(wx^{-1})$, where x, w are elements of an arbitrary Coxeter system (W, S) and l is the length function.

In §1 we find a combinatorial description of the first coefficient in q of the Kazhdan-Lusztig polynomial (Theorem A). In particular, for $x, w\in W$ satisfying $l(w)=l(x)+l(x^{-1}w)$ $(l(w)=l(x)+l(wx^{-1}))$, the first coefficient in q is equal to $c^-(x, w)-g(x^{-1}w)$ (resp. $c^-(x, w)-g(wx^{-1})$), where $c^-(x, w)$ $(g(w))$ is the number of coatoms (resp. atoms) of the interval $[x, w]$ (resp. $[e, w]$, e is the identity element) in the Bruhat order (see Def. 1.3).

In §2 we give the proof of the non negativity of $c^-(x, w)-g(x^{-1}w)$ for $x, w\in W$ satisfying $l(w)=l(x)+l(x^{-1}w)$.

Let us give a brief review of known results. It is conjectured in [KL] that all coefficients of the Kazhdan-Lusztig polynomials are non negative. This is still an open problem, but some of the special cases are verified. For example, this conjecture is correct for finite Weyl groups, affine Weyl groups and dihedral groups. M. Dyer has proved the non negativity of the first coefficients in q of the Kazhdan-Lusztig polynomials for $e, w\in W$ by showing that the first coefficient is equal to $c^-(e, w)-g(w)$ in this case and it is non negative ([D]). So, our results include his.

§1. Combinatorial description of the first coefficient.

At first, we shall define the Bruhat order and the Kazhdan-Lusztig polynomials. Throughout this article, (W, S) is an arbitrary Coxeter system, where S denotes a

Received December 17, 1992