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The Nullity of Compact Minimal Real Hypersurfaces in a Complex Projective Space

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Dedicated to Professor Tsunero Takahashi on his 60th birthday

1. Introduction.

Let M be a compact minimal submanifold of a Riemannian manifold $\tilde{M}$. Then the general situation of the problem we will investigate in this paper is the following:

PROBLEM 1.1. Estimate the index or the nullity of M from below, and furthermore determine M whose index or nullity attains the lowest bound in the estimation.

In general, this problem is very difficult, so it seems to be interesting and important to observe for specific ambient manifolds. Concerning with this problem, the following two results are well-known:

EXAMPLE 1.2 (J. Simons [S]). In the case where $\tilde{M}$ is the n-dimensional sphere and dimension of M is p, then the inequalities $\mathrm{ind}(M) \geq n-p$ and $\mathrm{nul}(M) \geq (p+1)(n-p)$ hold. Moreover, in each inequality, the equality holds only when M is totally geodesic.

EXAMPLE 1.3 (Y. Kimura [K]). In the case where $\tilde{M}$ is the complex projective space with complex dimension n and M a Kähler submanifold with complex dimension p, then inequality $\mathrm{nul}(M) \geq 2(p+1)(n-p)$ holds and equality holds only when M is totally geodesic.

Here $\mathrm{ind}(M)$ and $\mathrm{nul}(M)$ stand for the index and the nullity of M respectively. Moreover Ohnita [O, section 6] has obtained a generalization of their results above to the case of compact rank-1 symmetric spaces.

The purpose of this paper is to investigate the problem (1.1), mainly for the nullity, in the case where $\tilde{M}$ is the n-dimensional complex projective space P^nC with the Fubini-Study metric of constant holomorphic sectional curvature 4, and M a compact oriented minimal real hypersurface in P^nC. It should be noted that in the examples above the minimal submanifold with minimum index or nullity is always totally

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