TOKYO J. MATH.
VOL. 17, No. 1, 1994

Mean Value Results for the Non-Symmetric Form of the Approximate Functional Equation of the Riemann Zeta-Function

Isao KIUCHI

Keio University

(Communicated by Y. Maeda)

1. Statement of results.

Let $s=\sigma+it$ $(0\leqq\sigma\leqq 1,\ t\geqq 1)$ be a complex variable, $\zeta(s)$ the Riemann zeta-function, $d(n)$ the number of positive divisors of the integer n, γ the Euler constant and $\exp(2\pi i\alpha)=e(\alpha)$. We first define

$$R(s;\, t/2\pi)=\zeta^2(s)-\sum_{n\leqq t/2\pi} d(n)n^{-s}-\chi^2(s)\sum_{n\leqq t/2\pi} d(n)n^{s-1}\,,$$

where

$$\chi(s)=2^s\pi^{s-1}\sin(\pi s/2)\Gamma(1-s)\,.$$

As for this function $R(s;\, t/2\pi)$, Motohashi (see (1) of [6]) proved the following "weak form" of the Riemann-Siegel formula for $\zeta^2(s)$:

$$\chi(1-s)R(s;\, t/2\pi)=(t/2\pi)^{-1/4}\sum_{n=1}^{\infty} d(n)h(n)n^{-1/4}\sin(2\sqrt{2\pi tn}+\pi/4)$$
$$+O(t^{-1/2}\log t)\,, \tag{1.1}$$

where

$$h(n)=(2/\pi)^{1/2}\int_0^\infty (y+n\pi)^{-1/2}\cos(y+\pi/4)dy\,.$$

Kiuchi and Matsumoto (see Theorem 1 of [3]) started from this formula, and proved an asymptotic formula for the mean square of $|R(1/2+it;\, t/2\pi)|$:

$$\int_1^T |R(1/2+it;\, t/2\pi)|^2dt=\sqrt{2\pi}\left\{\sum_{n=1}^{\infty} d^2(n)h^2(n)n^{-1/2}\right\}T^{1/2}+O(T^{1/4}\log T)\,. \tag{1.2}$$

Received December 3, 1992