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Examples of Non-Einstein Yamabe Metrics with Positive Scalar Curvature

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Let M be a compact C^∞-manifold with $n=\dim M\geq 3$. For any Riemannian metric g on M, we denote its scalar curvature by S_g, and its volume form by dV_g. Yamabe [9] considered the problem of finding a metric which minimizes the functional $I(g):=\int_M S_g dV_g/(\int_M dV_g)^{(n-2)/n}$ in a given conformal class. Such a metric is called a *Yamabe metric* and has constant scalar curvature. This problem was solved completely by Schoen [7], and we know that there is a Yamabe metric in any conformal class. Conversely, a metric g with constant scalar curvature is a Yamabe metric, if $S_g\leq 0$ or g is an Einstein metric ([5]). The Yamabe metrics conformal to $S^1(r)\times S^{n-1}(1)$ are also known in explicit form ([2], [3], [8]).

In this paper, we give a sufficient condition for a metric to be a Yamabe metric, and examples of non-Einstein Yamabe metrics with positive scalar curvature.

THEOREM. *Let g be a Yamabe metric on a compact C^∞-manifold M with $S_g>0$, h a metric on M with constant scalar curvature, and φ a diffeomorphism of M such that $dV_{\varphi^*h}=\gamma dV_g$ for some number γ. If $\varphi^*h\leq(S_g/S_h)g$, then h is also a Yamabe metric. Moreover, if $\varphi^*h<(S_g/S_h)g$, then h is a unique Yamabe metric (up to a homothety) in the conformal class $[h]$ of h.*

REMARK. For any two metrics g and h, there is a diffeomorphism φ such that $dV_{\varphi^*h}=\gamma dV_g$ for some γ (see [4]).

PROOF. It suffices to show the case when $\varphi=id$. For any metric $\tilde{h}=u^{4/(n-2)}h\in[h]$, we have

$$I(\tilde{h})=\frac{\displaystyle\int_M (a_n|\nabla_h u|^2+S_h u^2)dV_h}{\left(\displaystyle\int_M u^p dV_h\right)^{2/p}},$$

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