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# Irregularity of Quintic Surfaces of General Type

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## Introduction.

Let $X$ be a hypersurface in $\boldsymbol{P}^3$ of degree $d$ defined over an algebraically closed field $k$ of characteristic 0. For $d \leq 4$, singularities on $X$ and properties of the resolution $\tilde{X}$ of $X$ have been studied. For example, if $X$ is normal, then it is known that $\tilde{X}$ is birationally equivalent to one of the following surfaces:

$d=1, 2$: a rational surface;

$d=3$: a rational surface or an elliptic ruled surface;

$d=4$: a $K3$ surface, a rational surface, an elliptic ruled surface or a ruled surface over a curve of genus 3.

(The case of $d=1$ or 2 is clear. For $d=3$, see Hidaka-Watanabe [3], and for $d=4$, Umezu [8]. The argument in [8] can also be applied to the case of $d \leq 3$.)

On the other hand, not many things are known about the case of higher $d$. The purpose of this paper is to prove the following

MAIN THEOREM. *Let $X$ be a normal quintic surface and $\tilde{X}$ denote its resolution. If $\tilde{X}$ is of general type, then its irregularity $q(\tilde{X})$ vanishes.*

REMARK. As we see in the following example, this result is not available for $d \geq 6$.

EXAMPLE (Zariski). Let $(X_0 : X_1 : X_2 : X_3)$ be homogeneous coordinates of $\boldsymbol{P}^3$ and put

$$X = \{X_3^6 - (F(X_0, X_1, X_2)^2 + G(X_0, X_1, X_2)^3) = 0\}$$

where $F$ and $G$ are homogeneous polynomials of degree 3 and 2 respectively. Then the irregularity of a resolution $\tilde{X}$ of $X$ is positive ([13]). The singularity of $X$ corresponds to the singularity of the curve $C = \{F(X_0, X_1, X_2)^2 + G(X_0, X_1, X_2)^3 = 0\} \subset \{X_3 = 0\} \simeq \boldsymbol{P}^2$. If $F$ and $G$ are general, the singularity of $C$ is at the six points of $\{F(X_0, X_1, X_2) = 0\} \cap \{G(X_0, X_1, X_2) = 0\}$ and each corresponding singular point on $X$ is defined locally by

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