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The Theta-Curve Cobordism Group Is Not Abelian

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Introduction.

A spatial theta-curve $f:\theta\to S^3$ is an embedding of a theta-curve with its vertices and edges labelled. Given two spatial theta-curves f and g, we can define a new spatial theta-curve $f\# g$, the vertex connected sum of f and g, up to ambient isotopy [7]. K. Taniyama [6] defines cobordism between spatial theta-curves, and observes that (1) the cobordism classes form a group Θ under vertex connected sum: the cobordism inverse of a theta-curve f is represented by the reflected inverse $f!$ of f; (2) a theta-curve is slice if and only if an associated 2-component parallel link is slice (i.e. bounds disjoint disks in the 4-ball). He investigates the theta-curve cobordism group Θ through constituent knots of theta-curves, but the following fundamental question is left open in [6].

QUESTION 1. *Is Θ an abelian group?*

This note presents an example answering the question in the negative. The proof consists of showing that certain 2-component links are not slice using the refinement of the Casson-Gordon technique due to P. Gilmer [2].

Finally we raise intriguing questions below.

QUESTION 2. (1) *Does Θ contain the free group of infinite rank?*
(2) *What is the center of Θ?*

1. Statement of results.

We use the same notation as in [6], e.g. i-th parallel link $l_i(f)$, reflected inverse $f!$ of a spatial theta-curve f, theta-curve cobordism group Θ. Given a knot K and $q\in\mathbf{R}$, $\sigma_{(q)}(K)$ is the signature of the matrix $(1-e^{2\pi iq})V+(1-e^{-2\pi iq})V^T$ where V is a Seifert matrix for K.

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