TOKYO J. MATH.
VOL. 17, NO. 1, 1994

A Construction of Irreducible Representations of the Algebra of Invariant Differential Operators on a Homogeneous Vector Bundle and Its Applications

Shin-ichi AZUMA

Waseda University

(Communicated by Y. Shimizu)

1. Introduction.

In this paper we construct irreducible representations of the algebra of invariant differential operators on a homogeneous vector bundle, and give a condition for the nontriviality.

Let G be a connected real semisimple Lie group with finite center, and let K be a maximal compact subgroup of G. Let E_τ denote the homogeneous vector bundle over G/K associated to a representation (τ, V_τ) of K (See [10] for the definition), and let $D(E_\tau)$ denote the algebra of G-invariant differential operators on E_τ.

Let (τ^*, V_{τ^*}) be the contragredient representation of τ, and let $\mathfrak{g}$ be the Lie algebra of G. Then for an irreducible $(\mathfrak{g}, K)$-module W (cf. §3), $\Gamma_\tau(W)=\mathrm{Hom}_K(V_{\tau^*}, W)$ gives rise to an irreducible representation of $D(E_\tau)$ unless trivial (Proposition 4.2). Our choice of W is rather special. But this specialization allows us to apply the classification theory of Langlands which tells us whether $\Gamma_\tau(W)$ be trivial or not.

The construction and the non-triviality condition (cf. §4) yield two applications: first, a simple proof of (a part of) theorem of Deitmar ([2, Theorem 6]) which claims that $D(E_\tau)$ is commutative if and only if $\tau|_M$ is multiplicity free (Proposition 6.2); second, a sufficient condition for some kind of restriction of the Poisson transform to be injective modulo the kernel of some surjection (Theorem 7.2). Because we can describe the injectivity condition in terms of Harish-Chandra's C-function (cf. §7), Theorem 7.2 is a weak analogue of the result for a line bundle (cf. [8]) which shows that the Poisson transform is injective if and only if C-function is non-zero.

The author should like to express his gratitude to Professor Y. Shimizu and Professor K. Minemura for their useful suggestions and encouragement. The author also thanks

Received October 26, 1992
Revised May 18, 1993