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Finite Type Minimal 2-Spheres in a Complex Projective Space

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§1. Introduction.

Let M be a compact C^∞-Riemannian manifold, $C^\infty(M)$ the space of all smooth functions on M, and Δ the Laplacian on M. The Δ is a self-adjoint elliptic differential operator acting on $C^\infty(M)$, which has an infinite discrete sequence of eigenvalues:

$$\mathrm{Spec}(M)=\{0=\lambda_0<\lambda_1<\lambda_2<\cdots<\lambda_k<\cdots\uparrow\infty\}.$$

Let $V_k=V_k(M)$ be the eigenspace of Δ corresponding to the k-th eigenvalue λ_k. Then V_k is finite-dimensional. We define an inner product $(\ ,\ )$ on $C^\infty(M)$ by

$$(f,g)=\int_M fg\,dV,$$

where dV denotes the volume element on M. Then $\sum_{t=0}^\infty V_t$ is dense in $C^\infty(M)$ and the decomposition is orthogonal with respect to the inner product $(\ ,\ )$. Thus we have

$$C^\infty(M)=\sum_{t=0}^\infty V_t(M)\qquad(\text{in } L^2\text{-sense}).$$

Since M is compact, V_0 is the space of all constant functions which is 1-dimensional.

Let $\tilde{M}$ be a compact C^∞-Riemannian manifold, and assume that M is a submanifold of $\tilde{M}$ which is immersed by an isometric immersion φ. We have the decomposition

$$C^\infty(\tilde{M})=\sum_{s=0}^\infty V_s(\tilde{M})\qquad(\text{in } L^2\text{-sense})$$

with respect to the Laplacian $\Delta_{\tilde{M}}$ of $\tilde{M}$. We denote by φ^* the pull-back, i.e., φ^* is an $\boldsymbol{R}$-linear map of $C^\infty(\tilde{M})$ into $C^\infty(M)$ such that

$$(\varphi^*F)(p)=F(\varphi(p)),\qquad p\in M,\quad F\in C^\infty(\tilde{M}).$$

For each integer s, $\varphi^*V_s(\tilde{M})$ is a subspace of $C^\infty(M)$. Then we have a decomposition

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