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Reidemeister Torsion of Seifert Fibered Spaces for $SL(2; C)$-Representations

Teruaki KITANO

Tokyo Institute of Technology
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§0. Introduction.

This paper is devoted to the study of the Reidemeister torsion. It is a piecewise linear invariant for n-dimensional manifolds and originally defined by Reidemeister, Franz and de Rham. In 1985 Casson defined an interesting topological invariant of homology 3-spheres by making use of a beautiful construction on the space of $SU(2)$-representations of the fundamental group. Later Johnson developed a similar theory of Casson's one by using the Reidemeister torsion as its essential ingredient. He also derived an explicit formula for the Reidemeister torsion of Brieskorn homology 3-spheres for $SL(2; C)$-irreducible representations. In this paper, we call this type Reidemeister torsion the $SL(2; C)$-torsion following Johnson. Let M_n be a 3-manifold obtained by the $1/n$-surgery on a torus (p, q)-knot. It is a Brieskorn homology 3-sphere $\Sigma(p, q, pqn \pm 1)$. The fundamental group $\pi_1 M_n$ admits a presentation as follows;

$$\pi_1 M_n = \langle x, y \mid x^p = y^q, ml^n = 1 \rangle$$

where m is a meridian of the torus knot which is a word of x and y and l is similarly a longitude. Johnson proved the following theorem.

THEOREM (Johnson). *The distinct conjugacy classes of the $SL(2; C)$-irreducible representations of $\pi_1 M_n$ are given by $\rho_{(a,b,k)}$ such that*

(1) $0 < a < p$, $0 < b < q$, $a \equiv b \bmod 2$,
(2) $0 < k < N = |pqn + 1|$, $k \equiv na \bmod 2$,
(3) $\operatorname{tr} \rho_{(a,b,k)}(x) = 2\cos \pi a/p$,
(4) $\operatorname{tr} \rho_{(a,b,k)}(y) = 2\cos \pi b/q$,
(5) $\operatorname{tr} \rho_{(a,b,k)}(m) = 2\cos \pi k/N$.

In this case the $SL(2; C)$-torsion $\tau_{(a,b,k)}$ for $\rho_{(a,b,k)}$ is given by

$$\tau_{(a,b,k)} = \begin{cases} 2(1-\cos \pi a/p)(1-\cos \pi b/q)(1+\cos \pi kpq/N) & a \equiv b \equiv 1,\ k \equiv n \quad \bmod 2 \\ 0 & a \equiv b \equiv 0 \text{ or } k \not\equiv n \quad \bmod 2\,. \end{cases}$$

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