

Parallelogram Tilings and Jacobi-Perron Algorithm

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0. Introduction.

The most simple example of quasiperiodic tilings is known as follows (See [1]). Consider a line $l(\alpha) = \{(x, y) \mid x + \alpha y = 0\}$ for each $\alpha \in [0, 1)$. Let $C(\alpha)$ be the set of squares (translates of the fundamental square) that l intersects, and let $S(\alpha)$ be the path along one side of the boundary of $C(\alpha)$. See Figure 1.

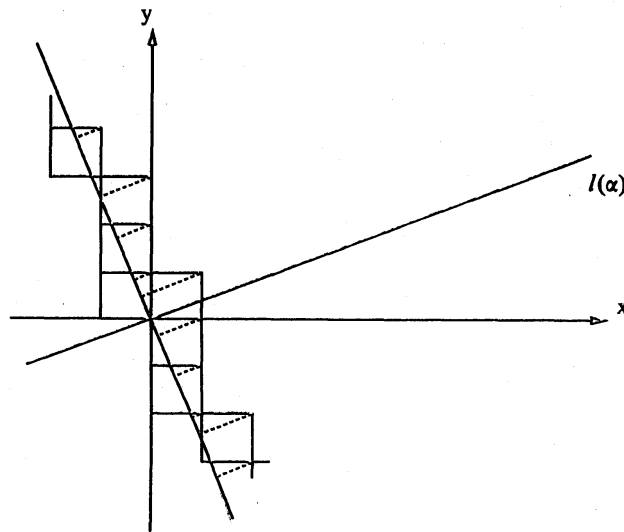


FIGURE 1. Figure of $l(\alpha)$ and projecting images of the vertical and horizontal edges along $(1, \alpha)$

If we project the stepped curve $S(\alpha)$ onto $l(\alpha)$ along the vector $(1, \alpha)$ then the images of the vertical and horizontal edges in $C(\alpha)$ form a tiling of $l(\alpha)$. We claim that the tiling is quasiperiodic iff α is irrational. We know also the generating method of the stepped curve $S(\alpha)$ by using the continued fraction algorithm and by introducing the substitutions of edges. In this paper we consider the stepped surface of a plane in \mathbb{R}^3