TOKYO J. MATH.
VOL. 17, No. 1, 1994

On the Topology of Simple Fold Maps

Kazuhiro SAKUMA

Tokyo Institute of Technology
(Communicated by S. Suzuki)

§1. Introduction.

The purpose of this note is to study the topology of simple fold maps of a closed n-manifold into a p-manifold $(n>p)$ and special generic maps of a closed orientable 4-manifold into an orientable 3-manifold. We call a smooth map f with only fold singularities a *fold map* and it is said to be *simple* if the connected component of $f^{-1}(f(x))$ containing x intersects $S(f)$ only at x for every $x \in S(f)$. *Special generic maps* are smooth maps which have only definite fold sigularities. Precise definitions of fold maps, simple fold maps and special generic maps are given in §2.

The results here extend and depend upon some related results in the author's previous paper [13], where special generic maps of simply connected $2n$-manifolds into $\boldsymbol{R}^3$ were studied. For a map $f: M^n \to N^p$, $S(f)$ denotes the set of the singular points of f, and when $n-p+1$ is even, $S^+(f)$ denotes the set of the fold points with even index and $S^-(f)$ the set of the fold points with odd index.

One of our main results is the following

THEOREM A. *Let M^n be a closed n-manifold and N^p a p-manifold ($n-p$: odd, $n>p$). Let $f: M^n \to N^p$ be a simple fold map. Then we have*

$$\chi(M^n)=\chi(S^+(f))-\chi(S^-(f)),$$

where χ denotes the Euler characteristic.

When $N^p=\boldsymbol{R}^p$, the above theorem has been obtained by Fukuda [5] without the assumption that f be simple. It is interesting that the topology of N^p does not affect the equality involving the Euler characteristics even if we replace $\boldsymbol{R}^p$ by any p-manifold in Fukuda's theorem, provided that f is simple.

In [13], for a special generic map f of a simply connected $2n$-manifold M^{2n} into $\boldsymbol{R}^3$ we have proved that $\chi(M^{2n})=2\#S(f)$, twice the number of connected components of $S(f)$ (when $2n=4$, $\#S(f)=\frac{1}{2}b_2(M^4)+1$). In the case $2n=4$, if we remove some restrictions on the source and the target manifolds, we have a weaker result on the

Received September 7, 1992
Revised October 6, 1993