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A Note on L^2 Harmonic Forms on a Complete Manifold

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Introduction.

In this note, we shall show a nonexistence result for harmonic forms with values in a vector bundle equipped with a Riemannian structure over a complete manifold. Moreover in relation with the result, we shall construct examples of harmonic mappings of finite total energy. This note is motivated by a recent paper of Elworthy and Rosenberg [5].

Vanishing theorems and growth properties for such forms (including harmonic mappings for example) have been investigated extensively by many authors from various points of views. Donnelly and Xavier [4] studied, for instance, the spectrum of the Laplacian acting on the square integrable forms on a negatively curved manifold and showed a sharp lower bound for the spectrum under a certain pinching condition on curvature. Their result gives in particular vanishing of L^2 harmonic forms. An integral identity on differential forms, (1.1) in Section 1, plays a crucial role in their paper. We remark that this formula was also obtained by Karcher and Wood [8] to study the growth properties for harmonic forms. In this note, we shall derive a consequence of the formula, which is stated in the following:

THEOREM 1. *Let M be a complete Riemannian manifold of dimension m, and let E be a real vector bundle endowed with a Riemannian structure. Suppose M possesses a pole o (a point at which the exponential mapping induces a diffeomorphism). Then there are no nontrivial square integrable, E-valued harmonic q-forms ($q=p$ or $m-p$) for a positive integer p less than $m/2$ if the radial curvature K_r of M satisfies either of the following conditions*:

$$-\left(\frac{m-p-1}{p}\right)^2 \le K_r \le -1 , \tag{1}$$

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