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# 2-Type Surfaces in $S_1^3$ and $H_1^3$

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## 1. Introduction.

In a series of recent papers ([4], [5], [14]) the technique of finite type immersions (see [9] for details) has been sistematically used to characterize certain interesting families of Riemannian submanifolds. The authors have used these arguments to try to classify surfaces satisfying certain characteristic differential equations in the Lorentzian space forms (see [2], [3] and [12]). It is well known that the shape operator of a pseudo-Riemannian surface does not need to be diagonalizable; because of this fact there are substantial differences between the definite and indefinite cases. Actually, it is possible to find a wide family of examples of surfaces in indefinite space forms having no Riemannian counterparts; the $B$-scrolls ([10] and [13]) and the complex circles ([16]) are some of these examples.

The finite type immersion tool allows to discover certain hidden facts in non flat Lorentzian ambient spaces $\bar{M}_1^3(c)$, with $c=\pm 1$. For instance, a totally umbilical surface does not need to be of 1-type; however both conditions are equivalent if and only if the surface is non flat. Actually, the following two quite interesting facts can be obtained from the pseudo-Riemannian version of Takahashi's theorem ([6] and [17]): (i) a surface in $\bar{M}_1^3$ is of 1-type if and only if it is either minimal or non flat totally umbilical in $\bar{M}_1^3$; and (ii) there exist flat totally umbilical surfaces in $\bar{M}_1^3$ which are *biharmonic*, i.e. its mean curvature vector field is harmonic, and therefore they are of infinite type.

On the other hand, $B$-scrolls as well as complex circles come out as surfaces in $\bar{M}_1^3$; it seems then reasonable to try to characterize them according to its finite type character. It should be noticed that $B$-scrolls already appeared in studying surfaces satisfying the condition $\Delta H=\lambda H$ in Lorentzian space forms.

In a more general situation, we look for 2-type isometric immersions into $\bar{M}_1^3$. The equation $\Delta H=\lambda H$ allows to reach only up to surfaces of 2-type with a zero eigenvalue

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