

Some Characterizations of Bloch Functions on Strongly Pseudoconvex Domains

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Introduction.

The purpose of this paper is to characterize Bloch functions on smoothly bounded strongly pseudoconvex domains in terms of invariant geometry, Bergman-Carleson measures and Kähler diffusion processes.

We will begin with describing the motivation of this paper. As is well known, the class of Bloch functions on the unit disc can be characterized in many different ways, and therefore it arises in several different areas such as function theory, operator theory and harmonic analysis etc. For the case of several complex variables, Timoney [21] extended a number of characterizations of Bloch functions on the unit disc to bounded homogeneous domains (see [21]). Later, Krantz and Ma [10] studied systematically Bloch functions on strongly pseudoconvex domains, and characterized them in terms of Schlicht disks, BMOA functions and normal families of mappings.

Recently, two new characterizations of Bloch functions on the open unit ball in C^n were given by Choa, Kim and Park ([5]) and by Muramoto ([18]); first, a Bergman-Carleson measure characterization ([5, Main Theorem]), and secondly a characterization in terms of hyperbolic Brownian motion ([18, Theorem]). (In [18], Muramoto assumed that $n=1$.)

It must be noted that Lyons pointed out already in [13] a close connection between hyperbolic Brownian motion and Bloch functions on the unit disc. Furthermore, using this connection, Lyons [13] proved a certain law of the iterated logarithm for boundary behavior of Bloch functions on the unit disc, which is regarded as a probabilistic analogue of Makarov's celebrated law of the iterated logarithm for radial behavior of Bloch functions on the unit disc ([15]). Moreover, in [13] it was proved that his probabilistic analogue implies Makarov's law of the iterated logarithm.

Now we will explain our results. Our main theorem is Theorem 2 stated in Section 2, which extends the characterizations by Choa, Kim and Park [5] and Muramoto [18]