TOKYO J. MATH.
VOL. 17, NO. 2, 1994

A Dehn Surgery Formula for Walker Invariant on a Link

Katsuhiro ISHIBE

Tokyo Metropolitan University
(Communicated by M. Sakai)

0. Introduction.

In 1985, Andrew Casson defined an integer valued invariant $\lambda(M)$ for any oriented integral homology 3-sphere M, which counts the "signed" irreducible representations of the fundamental group $\pi_1(M)$ into $SU(2)$ [1]. In 1989, Kevin Walker extended the Casson's invariant to rational homology 3-spheres, by taking into account the reducible representations of $\pi_1(M)$ coming from torsion [6]. In this paper we give a formula for Walker's invariant in the case where a rational homology 3-sphere H is obtained by Dehn surgery on a link L in a rational homology 3-sphere M, and furthermore the linking number between every pair of components of L is zero. In this case the Walker's invariant, $\lambda(H)$, can be expressed in terms of $\lambda(M)$, the surgery coefficients of L, a certain coefficient from each of the Conway polynomials of L and all its sublinks, and a certain function τ which was introduced by Walker. In the case of original Casson's invariant, a formula for Dehn surgery on a link in an integral homology 3-sphere was given by Jim Hoste [3]. We adapt his method to the case of the Walker's invariant and obtain a formula.

Suppose $L=\{K_1, \cdots, K_n\}$ is a link in a rational homology sphere M. Let $N(K_i)$ be a tubular neighborhood of K_i. Let $x_i \in H_1(\partial N(K_i); \boldsymbol{Z})$ be a primitive homology class. We call pairs $\{(K_1, x_1), \cdots, (K_n, x_n)\}$ *a framed link* and denote by $\chi((K_1, x_1), \cdots, (K_n, x_n); M)$, or simply by $\chi(L; M)$, the manifold obtained from M by Dehn surgery along L according to the given framings x_is. Let $\langle \cdot, \cdot \rangle$ denote the intersection pairing on $H_1(\partial N(K_i); \boldsymbol{Z})$. (The orientation of $\partial N(K_i)=\partial(M-N(K_i))$ is induced from that of $M-N(K_i)$ via the "inward normal last" convention.) Let m_i and l_i be the meridian and longitude of K_i respectively. Walker gives the following formula for Dehn surgery on a knot K (i.e. one component link):

$$\lambda(\chi(K; M))=\lambda(M)+\tau(m, x; l)+\frac{\langle m, x\rangle}{\langle m, l\rangle\langle x, l\rangle}\Gamma(K; M)\,.$$

Received March 5, 1993