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Geometry of Submanifolds in Terms of Behavior of Geodesics

Sadahiro MAEDA and Koichi OGIUE

Shimane University and Tokyo Metropolitan University

1. Introduction.

It is interesting to study Riemannian submanifolds M of a Riemannian manifold $\tilde{M}$ in terms of the behavior of geodesics of M. From this point of view, a totally geodesic submanifold is considered to be the simplest one. The second simplest one will be a *circular geodesic* submanifold, where every geodesic of M is a circle in $\tilde{M}$. In the case where $\tilde{M}$ is a real space form, a circular geodesic submanifold has parallel second fundamental form.

We recall the notion of *isotropic* immersions introduced by O'Neill ([13]): Let σ be the second fundamental form of M in $\tilde{M}$. Then the immersion is said to be isotropic at $x \in M$ if $\| \sigma(X, X) \| / \| X \|^2$ is constant at x. If the immersion is isotropic at every point, then there exists a function λ on M defined by $x \to \| \sigma(X, X) \| / \| X \|^2$ and the immersion is said to be λ-isotropic or, simply, isotropic.

It is known that "circular geodesic" always implies "isotropic" and that the class of isotropic submanifolds is too wide to classify. The first author studied characterizations of circular geodesic submanifolds of a sphere among isotropic immersions. It is reasonable to study some class of submanifolds between "circular geodesic" and "isotropic".

The purpose of this paper is to characterize parallel immersions of a Cayley projective plane into a sphere among isotropic immersions and to show that complex hypersurfaces with parallel second fundamental form are characterized by a geometric condition weaker than "circular geodesic."

2. Preliminaries.

Let (M, g) be an n-dimensional Riemannian submanifold of an $(n+p)$-dimensional Riemannian manifold $(\tilde{M}, \tilde{g})$. We denote by ∇ and $\tilde{\nabla}$ the covariant differentiations on M and $\tilde{M}$, respectively. Then the second fundamental form σ of the immersion is defined

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