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Invariant Bilinear Forms for Heisenberg Group

Keisaku KUMAHARA

Tottori University

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1. Introduction.

Let G be a connected, simply connected nilpotent Lie group and $\mathfrak{g}$ be its Lie algebra. Irreducible unitary representations can be described by the orbit method due to A. A. Kirillov ([Ki]). Let f be an element of the dual space $\mathfrak{g}'$ of $\mathfrak{g}$. Let $\mathfrak{h}$ be a real polarization at f. Then f defines a one-dimensional representation τ_f of a subgroup $D=\exp(\mathfrak{g}\cap\mathfrak{h})$ of G. We can get an irreducible unitary representation $U^{f,\mathfrak{h}}$ of G, which is the induced representation from the representation τ_f of D. The unitary equivalence class of $U^{f,\mathfrak{h}}$ is independent of $\mathfrak{h}$ and depends only on the coadjoint orbit containing f. And any irreducible unitary representation of G is equivalent to one of $U^{f,\mathfrak{h}}$. Since $\mathfrak{h}$ is isotropic with respect to the alternative bilinear form $\varphi_f(X, Y)=f([X, Y])$, $X, Y\in\mathfrak{g}_{\boldsymbol{C}}$, f defines τ_f.

In the present paper we study the non-unitary representations of the Heisenberg group of $(2n+1)$-dimension. Irreducible unitary representations of the Heisenberg group are essentially parametrized by unitary characters of the center. V. S. Petrosyan ([P]) studied the irreducibility of non-unitary representations of the Heisenberg group of 3-dimension induced from non-unitary characters of the center. To prove the operator irreducibility he used the method of the invariant bilinear forms which was used in [GGV].

First we fix a real standard polarization $\mathfrak{h}$ at $f\in\mathfrak{g}'$ (see §2 for the definition of standard) and take a complex linear form $\Lambda\in(\mathfrak{g}')_{\boldsymbol{C}}$ on $\mathfrak{g}$ such that $\mathfrak{h}$ is isotropic with respect to φ_Λ. We define a representation τ_Λ of D by $\tau_\Lambda(\exp X)=\exp(\sqrt{-1}\Lambda(X))$, $X\in\mathfrak{g}\cap\mathfrak{h}$. And we define a non-unitary representation $U^{\Lambda,\mathfrak{h}}$ of G induced from τ_Λ. We realize it on the space $\mathscr{D}(G/D)$ of C^∞-functions on G/D with compact support. In our case if $f\neq 0$ on the center of $\mathfrak{g}$, then $\mathfrak{h}$ is abelian and $G/D\cong\boldsymbol{R}^n$. So we denote $\mathscr{D}(\boldsymbol{R}^n)$ by $\mathscr{D}_\Lambda^{\mathfrak{h}}$ as the representation space of $U^{\Lambda,\mathfrak{h}}$. Thus our object of study is a family of non-unitary representations $\{(U^{\Lambda,\mathfrak{h}}, \mathscr{D}_\Lambda^{\mathfrak{h}})\mid\mathfrak{h}$ is standard, $\Lambda\in(\mathfrak{g}')_{\boldsymbol{C}}\}$.

We get a necessary and sufficient condition for the existence of an invariant bilinear

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