TOKYO J. MATH.
VOL. 18, NO. 1, 1995

The Involutions of Compact Symmetric Spaces III

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Introduction.

In this part, we will construct certain fibrations intended for study of differential geometry, especially that of Riemannian submersions. Their base manifolds are polars and the fibres are centrioles of the meridians (See Section 4 for the review of these terms). They are listed in Table II, which include such significant examples as various twistor spaces of Calabi, Penrose and others as well as the Hopf fiberings; we will briefly explain more recent twistor spaces of R. Bryant [Br] in Section 8. As an application we will study and classify the simple graded algebras

$$\mathfrak{g}=\mathfrak{g}_{-2}+\mathfrak{g}_{-1}+\mathfrak{g}_0+\mathfrak{g}_1+\mathfrak{g}_2$$

of this type (7.1). That was done by S. Kaneyuki ([K], [KA]) already, but we will translate the problem into that of the compact form of $\mathfrak{g}$ (Prop. 7.6) and then describe it geometrically in terms of two tiers of fibrations (7.10); the relevant manifolds such as the fibre of one fibration are the fixed point sets of an involution acting on those of the other. The translation is so satisfactory it gives some insight into the gradation and the classification becomes a matter of picking up certain 2-tiered fibrations as Theorem 7.10 indicates (See 7.14 for the comparison with Kaneyuki's table, which agrees with our results). We return to the noncompact $\mathfrak{g}$ and interpret its gradation in terms of an affine symmetric space (7.12 & 13). We will establish basic facts (5.9, 5.11, etc.) in Section 5. We begin this part of the series with Section 4 in continuation of [N67] for the convenience of quoting items in it.

We thank Prof. Soji Kaneyuki warmly for valuable informations.

4. Definitions, notations and conventions.

4.1. We denote by $F(\tau, M)$ the fixed point set of a transformation τ of M; similarly, for a set S of transformations, $F(S, M)$ denotes the common fixed points of the members of S. *A symmetric space*, or just *a space*, is a manifold M with the (*point*) *symmetry* s_x

Received December 6, 1993