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Defining Ideals of Complete Intersection Monoid Rings

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In this note, we will extend Delorme's result about monomial curves [1] to Z^n-graded rings. To do this, we will define an ideal $I(V)$ associated with a submodule V of Z^N. It is generated by polynomials associated with vectors of V (see §1). And we have various examples of such ideals, e.g., defining ideals of monomial curves, that of Z^n-graded ring, and an ideal generated by 2×2 minors of a matrix. In general, $\mathrm{ht}\, I(V)=\mathrm{rank}\, V$, $I(V)$ is not necessarily prime, and we will give a condition that $I(V)$ is prime (Proposition 1.3).

In section 2, we will give the condition that $I(V)$ is a complete intersection ideal when V is contained in the kernel of a map $Z^p \to Z^q$ consisting of positive integers (Theorem 2.4). And we give a proof of the Delorme's result that any complete intersection monomial curve in A^r is induced by a complete intersection monomial curve in A^{r-1} (Corollary 2.5). We also show that if $\mathrm{rank}\, V<N-1$ and if $I(V)$ is a complete intersection, it is generated by a part of a minimal generating system of a complete intersection homogeneous ideal of height $N-1$ of the form $I(V')$ (Theorem 2.10).

1. Definitions and preliminaries.

Let $A=k[X_1, \cdots, X_N]$ be a polynominal ring over a field k. For $v\in Z^N$, we denote the i-th entry of v by $\sigma_i(v)$, and put

$$F_+(v)=\prod_{\sigma_i(v)>0} X_i^{\sigma_i(v)}$$

$$F_-(v)=\prod_{\sigma_i(v)<0} X_i^{-\sigma_i(v)}$$

$$F(v)=F_-(v)-F_+(v)\,.$$

(If $\sigma_i(v)<0$ for all i, we put $F_+(v)=1$. And if $\sigma_i(v)>0$ for all i, we put $F_-(v)=1$.) For a submodule V of rank r of Z^N with $0<r<N$, we define an ideal $I(V)$ of A generated

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